Managing Multiple Selling Channels in Technology Driven Markets

By

Hila Etzion

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Curriculum Vitae

The author was born in Jafa - Tel Aviv, Israel on 26 of December, 1974. She attended the Technion, Israel's Institute of Technology from 1995 to 1999, and graduated with a Bachelor (Summa Cum Laude) in Industrial Engineering and Management. She came to the University of Rochester in the summer of 2000 and began graduate studies in the Computers and Information Systems Department, W.E Simon Graduate School of Business Administration. She received a Xerox Fellowship in 2000, 2001, 2002 and 2003. The author pursued her research in economics of information systems under the direction of Professor Edieal Pinker and received the Master of Science degree in 2003.

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This work is dedicated to my best friend and advisor in life, my mother, Rachel Etzion, who did not live to see the end of it. This accomplishment is more hers than mine.

Abstract

In this dissertation I develop three game theoretic models that capture the interactions between buyers and sellers in B2C, B2B and software markets, when sellers can utilize multiple selling channels, and hence need to understand the relationships between demands on the different channels.

In the first model, presented in Chapter 3, I examine how sellers in B2C markets can use posted prices and online auctions in parallel to sell to heterogeneous consumers. I model consumer choice of channels, and thus market segmentation, and find that consumers who value the item for more than its posted price use a threshold policy to choose between the two channels. I explain how optimizing the design-parameters enables the seller to effectively segment the market so that the two channels reinforce each other and cannibalization is mitigated.

In Chapter 4, I model a B2B spot market with two supplier types: a supplier who faces contracted demand with fixed unit price, and a supplier who works solely on the spot market. I examine when the supplier that has contracts should use the spot market as an additional channel, which supplier type benefits more from the existence of the spot market, and which supplier type has a higher incentive to invest in extending the spot market. I study how the contracted demand affects the production decision and profit of the supplier with no contracts, and I show that the supplier that has contracts and buyer–firms benefit from negative correlation between the demands on the two channels.

In Chapter 5, I develop a conceptual and analytical model of the interaction between a base-software producer, ISVs selling specialized applications that run on the base-software,

and user firms, in a horizontally differentiated market. The model captures the tradeoffs user-firms face when choosing between in-house development of business applications and buying packaged applications. I show that as application development cost decreases, the base-software producer prefers having a network of ISVs rather than developing and selling integrated applications.

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Foreword

The work on the third chapter, "Simultaneous Use of Auctions and Posted Prices for Online Selling", was done in collaboration with Professor Edieal Pinker, and Professor Avi Seidmann. The model, the solution and the analysis of most of the results were the author's creative work. However, continuous feedback and guidance from Professors Edieal Pinker and Avi Seidmann significantly improved the quality of this research.

The work on B2B spot markets and forward contracts, presented in Chapter 4, was inspired by Professor Pinker's vision and ideas. The model of the problem studied, the solution and the analysis of the results were the author's creative work.

Chapter1

Introduction

With the proliferation of e-commerce and communication technology, firms have become more creative in their use of multiple selling channels for price discrimination and profit maximization. New channels, such as online auctions and online spot markets, are being used in parallel with more traditional selling channels, such as fixed prices and forward contracts. In this dissertation, I examine how buyers and sellers can take advantage of such new venues, and how they should adjust their relevant operating decisions.

Sellers in B2C (business to consumer) markets can choose between online auctions and posted prices for selling their goods online. A posted price segments consumers into those who value the item for more than the posted price and those who do not. In the absence of other selling channels, only consumers that value the item for more than its posted price buy the item. An auction, on the other hand, does not specify a price. The seller determines the auction length and the number of units offered, and consumers submit bids that depend on their valuations for the item. The consumers that submit the highest bids win the item. Hence, the auction price can vary from one auction to the other, depending on the number of consumers that arrive at the website, and their valuations for the item. While there is extensive literature on choosing a selling channel, auction or posted price for selling consumer goods, and on the optimal design of such a channel, we find only limited work that examines the seller's incentives for using the two channels in parallel, or models how buyers choose between the two channels, when offered in parallel. Our model of an online seller offering identical items using auctions and posted price at the same time, presented in Chapter 3, begins to fill this gap in the literature. Not only do we model how buyers choose between the two channels, but we also find the optimal dual-channel design for the online seller, and examine under which circumstances the dual channel can significantly increase the seller's profit when compared with a single channel, auction or posted price. In addition, we show the importance of managing the two channels jointly, and how independent design of each channel can result in losses when adding auctions parallel to a posted price.

In our model consumers arrive stochastically, at different times during the auction. Therefore, a consumer's choice of channel depends not only on the consumer's valuation for the item, but also on his arrival time. Using game theory we find consumers' bidding and auction participation strategies, and show that there is a unique symmetric equilibrium in which consumers who value the item for more than the posted price use a threshold policy to choose between the two channels.

We next examine sellers in B2B (business to business) markets. Such firms are the suppliers for buyer-firms, which usually have their own customers. The two most commonly used selling channels in B2B markets are forward contracts and spot markets. A forward contract, between a supplier and a buyer-firm, is signed before the buyer-firm knows its demand. The contract specifies the quantity to be delivered by the supplier to the buyer-firm, and the price that the buyer-firm would pay per unit. The literature distinguishes between "fixed quantity contracts" and "flexible quantity contracts," since the latter allow the buyer to order any quantity at the pre-agreed unit price, within some range. In addition, research on contracts assumes that the supplier incurs a penalty cost when he can not meet the contracted demand. This penalty cost can be specified in the contract or can be the result of losing future sales and damaged reputation.

Advances in communication networks and the widespread use of the Internet enable new online spot markets to proliferate in industries that used to be dominated by forward contracting. The spot market price is determined by the total demand (demand from all buyer-firms that work on the spot market) and the total supply on the spot market. Demand on the spot market originates from buyers who did not contract in advance, as well as from buyers whose demand exceeded the amount they contracted in advance. Supply on the spot market can originate from suppliers who over-estimated their contracted demand, as well as from suppliers who produce for the spot market. While the literature on B2B markets examines how buyer-firms should optimize using both forward contracts and spot markets, there is little research on how suppliers should operate in the presence of both channels. In addition, no previous research considers a market with two supplier types: suppliers that have forward contracts and suppliers that produce only for the spot market. Our research fills this gap in the literature. Since the buyers' side has been well studied, we do not model how buyers choose between the two channels or how they utilize both. Instead, we allow for both positive and negative correlation between spot market demand and contracted demand. We model a make-to stock industry with two suppliers of different types: one supplier that has contracts and one supplier that produces only for the spot market. We find the condition under which the supplier with contracts should satisfy his contracted demand before offering inventory on the spot market, and we show when this supplier should use the spot market as an additional channel and when he should use it only when contracted demand is low. We examine which supplier type benefits more from the existence of the spot market, the one that has forward contracts or the one that produces only for the market, and which supplier type has a higher incentive to invest in extending the spot market. We study the effect of the contracted demand on the production decision and profit of the supplier with no contracts, and answer whether this supplier prefers to compete with a supplier that does not have contracts. Analyzing the effect of the correlation between the contracted demand and the spot market demand on our results, we find that suppliers with contracts and buyer –firms benefit from negative correlation between the demands.

In the last model in this dissertation, presented in Chapter 5, I focus on the software industry and study the evolving market for independent software vendors (ISVs) that sell packaged business applications which operate in conjunction with base-software, such as database management systems (DBMS) and operating systems. The balance of power usually lies with the base-software producer, who controls the degree of openness of the interfaces to the base-software and how much information about the base-software and its interfaces to disclose to the ISVs. Hence, the base-software producer can control the number of ISVs in the market by offering subsidies or by charging fees. User-firms buy the base-software from the base-software producer, but can choose whether to develop a specialized application in-house, achieving perfect fit with their company needs, or buy a packaged application which might not exactly match their needs. User-firms are heterogeneous in their application needs and their scale of operation, and they incur a misfit cost when using an application that does not match their needs. I start by examining the tradeoffs user-firms face when choosing between the two channels: in house development and packaged application. Then, after deriving the user-firms' demand in each channel, I find the optimal strategy for the base-software producer. That is, I find under which conditions the base-software producer should support a channel of ISVs, under which conditions he should forgo such a channel and sell the base-software only to firms that can afford in-house development of business applications, and under which conditions he should develop and sell applications rather than support ISVs. I show that as application development cost decreases, the base-software producer's strategy changes from selling integrated applications to having a network of ISVs.

The three models presented in this dissertation examine interactions between buyers and sellers or between sellers of different types, when sellers have the choice of multiple selling channels and need to consider the relationship between demands on the different channels. The rest of the dissertation is organized as follows: In Chapter 2, I review the literature related to the three models. In Chapter 3, I study the simultaneous use of auctions and posted prices for selling consumer goods online. In Chapter 4, I examine how suppliers that have forward contracts and suppliers that produce only for the spot market interact, and I find the resulting spot market supply and industry supply. In Chapter 5, I model the interaction between a base-software producer and ISVs, when user-firms can choose between in-house development of business applications and buying packaged applications. At the end of each chapter the main conclusions are summarized, and longer proofs, which were not given in the body of the chapter, are presented. Chapter 6 concludes the dissertation, summarizing its contribution and giving directions for future research.

Chapter 2

Related Literature

The problems studied in this dissertation, cover three different literatures: the literature on auctions and on auctions versus posted prices in B2C (business to consumer) markets; the literature on B2B (business to business) spot markets and forward contracts; and the literature on platform software and applications. I review each of these literatures below in detail.

2.1 Auctions and Posted Price in B2C Markets

Auction markets have been of interest to researchers for generations. There exists an extensive literature that examines the optimal design of auctions and the ranking of different auction mechanisms: Milgrom (1987), McAfee and McMillan (1987), and Klemperer (1999) provide excellent surveys. Traditionally, analysis of auction design has focused on the auction mechanism itself. In these analyses, an auction is fully characterized by how bidder valuations are revealed and how the actual goods are allocated. Much of the focus has been on showing under which circumstances common auction mechanisms are equivalent and on proving when these mechanisms are truth-revealing. The widespread use of online auctions has brought a new set of managerial problems to the forefront that have yet to be studied. Pinker et al. (2003) survey the current state of research on the specific problems faced in the design of such auctions.

The problem of optimally selecting and designing a single selling mechanism, auction or posted price, has been well addressed. Wang (1993) considers the impact of the dispersion in the distribution of buyers' valuations on the choice between a posted price and an auction for a seller selling one unit of a good. He demonstrates that an auction is preferred when buyer valuations become more disperse. His model, like many others, ignores buyers' costs that are associated with auctions, such as the cost of delays and the cost of monitoring the auction. Wang does not model how consumers would choose between the two mechanisms, since he does not consider simultaneous use of both. Hence, the set of consumers is identical for both methods (it does not depend on the choice of mechanism). Harstad (1990) uses a model in which the seller's choice of selling method and reserve price does affect the number of bidders attending the auction. Ehrman and Peters (1994) consider a waiting cost for bidders due to the disappearance of outside alternatives. De Vany (1987) considers a seller with one unit of a commodity choosing between three mechanisms: an auction after a fixed time T, an auction after a fixed number of consumers have arrived, and posted price. Consumers incur a cost of waiting when an auction mechanism is chosen.

Some researchers have considered the choice of mechanism when the seller offers multiple homogenous units of the good (Arnold and Lippman (1995), Harris and Raviv (1981), Riley and Zeckhauser (1983), Maskin and Riley (1989)). Others have tried to explain the coexistence of the different selling mechanisms in a market, and examine the equilibrium of mechanisms in a competitive environment (Peters (1999), Epstein and Peters (1999), and Kultti (1997)). Recently, Gallien (2002) has compared the fixed price, dynamic posted price, and online auction mechanisms when selling multiple units to risk-neutral and time-sensitive consumers. Each buyer is characterized by his valuation of the item and his arrival time, and a buyer's net value decreases in the interval between his arrival and the time he obtains the item. Yet, though mechanism selection has been well covered, there is little research on how to operate and design such selling mechanisms in parallel. In addition, the economic benefits and limitations for a firm that concurrently employs multiple selling mechanisms are not clear.

Vakrat and Seidmann (1999) study simultaneous sales of identical products using online auctions and a fixed price catalog. Their empirical research shows that the auctions result in an average discount of 25% relative to the catalog prices. They model a one-unit English auction and assume that the number of bidders is deterministic, and that consumers have full information about the catalog price. They find that the expected auction price is a function of the number of bidders and of delay and search costs associated with the auction. Their paper does not model consumers' choice between the two channels, nor does it show what incentives the seller has to conduct such an auction. Van Ryzin and Vulcano (2002) examine the optimal pricing-replenishment policy when the firm sells in two markets, one fixed price market and one auction market, and demand comes from two different and independent streams of customers, so there is no need to model consumers' choice. In their model, the seller decides how to split the inventory between the two markets.

Another use of the two selling mechanisms simultaneously is the "buy now" price offered on many C2C auction sites. On Yahoo Auctions, for example, the auction will automatically close when a bidder meets the specified buy-now price, and the item is sold to that bidder. Another example is eBay's "Buy It Now" option, which is only shown on listings until an item receives its first bid, or, when the seller sets a reserve price, until the reserve is met. In this business model the auction is the main selling venue, and the buy-now price is the secondary channel. Budish and Takeyama (2001) model an English auction with a buy-now price. Their model has only two bidders and two possible types: a high-valuation and a low-valuation consumer. They show that the seller is strictly better off by adding a buy-now price to the auction only when bidders are risk-averse. This result, however, does not hold in a more general framework with N valuations. Hidvegi et al. (2002) model an auction with N bidders, having continuously distributed private valuations, and show that a bidder with a very high valuation compared to the buy price will use the buy price unconditionally, a bidder with a valuation close to the buy price will only use the buy price when the current bid reaches a threshold price, and there is no change in the optimal bidding strategy for a bidder with a valuation lower than the buy price. They find that when either party is risk-averse, a buy-price auction is strictly better for the seller. They do not consider delay costs for bidders. Reynolds and Wooders (2003) show that, when bidders are risk neutral, an auction with a buy-now price is revenue equivalent to the standard English ascending bid auction, so long as the buy-now price is not too low. When bidders are risk averse, however, auctions with buy-now prices are advantageous for the seller. They compare the seller's revenue and bidders' payoff for two auctions formats - the Yahoo format and the eBay format.

It is important to mention that, with the exception of Gallien (2002), the existing research does not model the fact that different consumers arrive at different stages of the online auction and that their expected utility from bidding is a function of their arrival time. Even those papers that associate a delay cost with bidding, assume that all bidders spend the same amount of time in the auction and thus incur the same delay cost.

Clearly, such assumptions are not suitable for the modeling of online auctions, which can last for as long as a week or more. Our model addresses this issue.

2.2 Spot Markets and Forward Contracts in B2B Markets

Current research on contracts and spot markets assume that buyers optimize on using both channels (risk management), and there is a tradeoff between contracting earlier, when demand is uncertain, for a known unit price, and waiting to buy on the spot market, after demand is realized, for an uncertain unit price. For example, Wu et al (2001, 2002) and Araman et al (2001) consider situations where the buyer reserves capacity before demand is known at a per unit price, and after demand is realized decides how much of the reserved capacity to utilize at an additional execution cost per unit and how much to purchase on the spot market. Deng (2002) considers a fixed quantity fixed price contract with the option for a supplementary purchase at a "spot" price. In his paper, the availability of spot supply is constrained by the supplier's production quantity, and the spot price is an exogenous random variable, bounded by the wholesale price. Unlike this previous work, which examines a buyer's strategic behavior and a single supplier, we model strategic behavior of competing suppliers in B2B spot markets, and generalize the demand side, allowing for both negative and positive correlation between spot market demand and contracted demand. In addition, in most of the existing literature, the spot price is modeled as an exogenous random variable, independent of the quantity traded by each of the agents. For examples see Wu et al. (2001, 2002, and 2003), Araman et al. (2001), Deng (2002) and Seifert et al. (2003). However, when the number of market participants is limited, as is true in many existing marketplaces, suppliers and buyers may have the power to influence the market. In expectation of this, they may change their decision at the initial contracting stage. Therefore, we model the expected spot-market price as a decreasing function of the quantity offered by each supplier.

Tunca (2002) and Lee and Whang (2002) consider markets which enable parties to readjust (buy and sell) their contracted positions to take advantage of updated information. Tunca models a monopolistic supplier who sells input to manufacturers of a consumer good. In the first period, contracting takes place. The contracts specify the quantity that will be delivered to each manufacturer at the third period, and the unit price.

In the second period, manufacturers receive signals regarding demand and can buy and sell the intermediate product on the B2B exchange. In the third period, consumer market clears with the quantities produced by the manufacturers. Tunca examines whether and to what extent trading on B2B exchanges, once it becomes prevalent, will make traditional supply chain contracting obsolete. He finds that the existence of the exchange reduces transaction prices between the supplier and the manufacturers. When the exchange is sufficiently liquid, parties choose not to engage in contracting at all, carrying out all purchases through the exchange. Lee and Whang (2002) consider a secondary market where resellers can trade between themselves. In this two-period model, resellers order from a single manufacturer at the beginning of the first period, then, after first period demand is realized, they can sell/buy on the secondary market based on the on-hand inventory and expectations for second period demand. The demand at the second period is independent on the demand in the first period. The price of the secondary market is the market clearance price. Seifert et al (2003) take into account spot price uncertainty, correlation between demand and spot prices and risk aversion. They analyze how spot markets affect the optimal order quantity via forward contracts, and asses the effect of risk-averse decision making on the optimal procurement strategy. In their model, the buyer can use the spot market to buy more units, when demand exceeds inventory on hand, and to sell excess inventory, in case demand turned out to be low. In either case, the actual spot price does not depend on the quantity traded by the firm on the spot market. In their model, if the buyer is risk neutral he would order an infinite capacity via forward contract whenever the expected spot price is higher than the contract price.

Several papers consider one period or multi-period flexible quantity contracts and a spot market. Araman and Ozer (2003) examine production and allocation decisions for a supplier who sells through two channels, a long term contract and spot market, where the contract defines the supplier's capacity for a multi-period finite horizon. At each period a variable "state of the world" determines the distributions of the spot price and the demand from the manufacturer for that period. At the beginning of each period the supplier observes the state of the world, the initial net inventory and the remaining production capacity, then, he decides how much to produce and how much to trade on the spot market in that period. These decisions determine the inventory available to satisfy the manufacturer's demand. If the manufacturer's demand exceeds the supplier's inventory, the excess demand is back-ordered to the next period and the suppliers incur a penalty cost.

Sethi et al. (2003) examine single and multi-period quantity flexible contracts, involving one demand forecast update in each period and a spot market. The contract permits the buyer to order at two points of time, once at the beginning of the period and once after obtaining a demand forecast, at which time the buyer may also purchase any amount of the product in the spot market. The contract specifies a limit to the second order quantity as percentage of the first order quantity.

While most of the reviewed work (with the exception of Araman and Ozer (2003)) assume that buyers optimize on using both channels, i.e., forward contracts and spot market, Caldentey and Wein (2004) assume that buyers split into two groups, based on the contracted price chosen by the seller. Buyers in the first group contract in advance and do not use the spot market, while buyers in the second group use only the spot market. Caldentey and Wein (2004) consider a single-product, make-to-stock manufacturer who uses two alternative selling channels: long term contracts and a spot market which consists of electronic orders (e-orders arrive stochastically; each order posts a price and the manufacturer decides if to accept it). Customers with reservation price lower than the contracted price wait for the spot market. Customers with reservation price higher than the contracted price split into two groups: *speculators*, who wait for the spot market, and *regular* buyers, who choose to form contracts. The percentage of speculators is increasing with the contracted price. The manufacturer's control problem is to select the optimal long-term contract price as well as the optimal production (i.e., busy/idle) and electronic-order admission (i.e., accept/reject) policies to maximize revenue minus inventory holding and backorder costs.

Our research differs from previous work, since we consider a market with two types of suppliers and examine how they affect each other's decisions and profits. To the best of our knowledge, this scenario has not been studied before.

2.3 Supply Chain Management in the Software Industry

Relatively few papers have analyzed the interaction between a base-software producer, independent software vendors (ISVs) selling specialized applications that run on the base-software, and end-user firms

Economides and Katsamakas (2004), develop a framework to characterize the optimal two-sides pricing strategy of a platform firm, that is, the pricing strategy towards the direct users of the platform as well as towards firms offering components that are complementary to the platform. They find the equilibrium prices for the platform, the application(s), and the platform access fee for applications (the application provider pays a per unit access fee to the platform firm, which is set by the platform firm). Their model has general demand functions, it allows for complementarities between the platform and each application, and users have a preference for application variety. They show that the platform firm subsidizes the application when the demand for the platform is stronger than the demand for the application, or the own-price effect of the platform is weak relative to the complementarily between the application and the platform. Our model, presented in Chapter 5, differs since we use spatial differentiation and derive the demand for applications from the user-firms' utility functions. In our model, firms are heterogeneous in operation scale and application needs, and they have the alternative of developing an application in-house. While Economides and Katsamakas (2004) examine one platform producer and N independent (non-competing) applications, our model allows for spatial competition between applications. In addition, we consider ISVs entry cost and find the equilibrium number of applications in the market. In our model the subsidy (fee) is a fixed amount and not a linear function of the number of applications sold.

This research is related to the literature on systems and complementary goods. Previous work examines the implications of compatibility, Matutes and Regibeau (1988) and Economides (1989), the effects of different ownership structures, Cournot (1838), Economides and Salop (1992), and Farrell and Katz, (2001), and the implications of bundling. For a summary of this literature see Economides and Katsamakas (2004). None of the above papers focus on the software industry. As in Economides and Katsamakas (2004), we assume that the components of the system (platform/base-software and application) are not symmetric. In our model, the base-software producer can prevent entry of firms selling applications by controlling the openness of the interfaces to the base-software. On the other extreme, the base-software producer can subsidize entry of firms selling applications (ISVs).

Our research examines when it is optimal for the base-software producer to integrate the applications and set both prices to maximize total profit. Thus, our work also relates to game theoretic research on channels and channels competition. This literature examines the competition between manufacturers under different assumptions on the vertical channels structure, and tries to answer whether a manufacturer should vertically integrate or completely decentralize its channel of distribution. Four models of channel interactions are considered in the literature, each with different numbers of channel members: (1) A single manufacturer and a single retailer channel (Jeuland and Shugan 1983); (2) Each manufacturer has an exclusive retailer, and there are two (or more) such exclusive pairs (McGuire and Staelin 1983); (3) Two manufacturers with a common retailer (Choi 1991, Sudhir 2001); (4) Two manufacturers with two competing retailers, each sells both products (Lee and Staelin 1997). Three vertical pricing games have been considered in this literature: Manufacturer Stackelberg, Retailer Stackelberg and vertical Nash. The optimal prices and profits of each channel member have been examined for each of these games (price leadership structures), and for different demand functions: linear or non-linear. For models with more than one manufacturer, effects of product differentiation and manufacturers cost differences on channel prices and profits have also been examined (Choi, 1991).

Kadiyali et al (2000) examine if these models can be used to measure channel power, which is defined as the proportion of channel profits that accrue to each of the channel members. They generalize Choi's model by allowing for a continuum of possible channel interactions between manufacturers and a retailer (not only three pricing games) and by allowing heterogeneity in manufacturer-retailer interactions.

Our model is tailored for the software industry and differs from previous work on channels and channels coordination. In the setting we examine, the base-software producer sets the price of the base-software to end users, and the ISVs (independent software vendors) set the prices of the applications. The only interaction between the base-software producer and an ISV can be via subsidy (or fees). In the traditional manufacturer-retailer setting, the manufacturer sets the wholesale price, charged from the retailer, and the retailer sets the price to end-users, often leading to double marginalization. When the manufacturer adds direct sales, end-users can choose whether to buy the product from the manufacturer or from the retailer, but they do not need to interact with both firms, as they do in our model. Another difference is that in our setting users-firms can develop the business application in-house. These special characteristics of software industry are not considered in the traditional supply chain management literature.

Chapter 3

The Simultaneous Use of Auctions and Posted Prices for Online Selling

3.1 Introduction

In the business to consumer market many firms are selling the same or almost identical products online using auctions and fixed prices simultaneously. The Internet enables firms to operate the two venues in the same space (the World Wide Web) and time and allows consumers to observe and compare the two selling channels with no additional costs. For example, IBM offers selected products via auctions on eBay while the same products are sold for posted prices on IBM's website; CompUSA conducts auctions for new and refurbished products on a dedicated auction website, while selling identical items for posted prices on its catalog web site; Sam's Club operates the two selling channels within the same website; airline tickets are sold for fixed prices as well as via reverse and forward auctions.

The practice of operating auctions and fixed prices in parallel, on the Internet, raises many important questions. Clearly the two selling channels cannot be treated independently. Optimizing each channel separately results in a suboptimal global design, as the two channels compete in the same market. Though the problem of optimally selecting and designing a single selling mechanism (auction or posted price) has been well addressed in the literature, there is little research on how to operate and design such selling mechanisms in parallel. In addition, the economic benefits and limitations of using both auctions and posted price are not clear. On one hand, an auction creates a venue for selling to consumers who do not value a product as much as the posted price, thereby increasing revenues. On the other hand, the auction channel may attract consumers who otherwise would have bought at the posted price, thereby reducing revenues. Our research focuses on the following issues:

- How do consumers behave when faced with the choice between the two channels?

- What is the optimal choice of the auction parameters, i.e., auction length and quantity, when identical items are sold for a posted price by the same outlet?
- What is the optimal posted price, when identical items are being auctioned?
- When does the dual channel outperform the single channel (posted price only)?

We develop a mathematical model that addresses these questions. In our model, a monopolist sells identical items using a sequence of sealed-bid second-price auctions and a posted price at the same time. Our research contributes to the existing auction literature by introducing the following three changes. First, the auctions are conducted parallel to a posted price and serve the same stream of consumers, so arriving consumers can choose between purchasing the item and bidding. Second, in our model the number of bidders is stochastic, and consumers can arrive at any time during the auction. Thus, different consumers spend different amounts of time in the auction and incur different delay costs, depending upon their arrival time. Finally, the auctioned quantity is an endogenous decision variable, set by the seller in order to maximize total (auction and fixed price) revenue.

Our results rely on a model of consumer behavior that defines how consumers choose between bidding and purchasing the item for its posted price, when the seller has an unlimited supply of the item. We prove that there exists a symmetric equilibrium in which consumers who value the item for more than its posted price, the "high valuation consumers," use a threshold policy to choose between the two selling channels. The threshold defines an upper bound on the remaining time of the auction: if the remaining time observed by a high valuation consumer upon his arrival exceeds the threshold, the consumer chooses to purchase the item for its posted price. If the remaining time is less than the threshold, the consumer chooses to participate in the auction. We also show that the optimal bidding strategy in the sealed-bid second-price auction is no longer truth-revealing. For a consumer who values the item for more than the posted price, bidding his true valuation is weakly dominated by placing a bid equal to the posted price.

We formulate a nonlinear optimization problem for choosing the posted price, the auction quantity, and the auction length when the seller's objective is to maximize expected revenue per unit time. Based on numerous numerical experiments, we argue that the optimal posted price in the dual channel is unique and is higher than the optimal posted price in the absence of auctions, and that a seller can significantly increase his revenue by adding an auction channel parallel to the posted price channel. Depending on the model's parameter values, we find that the optimization results in one of the following two strategies: (1) the seller should conduct one-unit auctions and decrease the auction length as the consumer arrival rate increases, or (2) the seller should conduct long auctions and increase the size of the auction lot as the consumer arrival rate increases. In the first strategy the number of units auctioned per consumer is small; in the second, this ratio is significantly higher. Which of these two strategies is optimal for the seller depends on the consumer arrival rate and the delay cost per unit time incurred by high valuation consumers. Our results confirm that the seller's revenue from the dual channel can be higher than the optimal revenue achieved by using a posted price alone, or by managing the two channels independently.

The chapter's structure is as follows. In §3.2, we consider a monopolist seller and construct a model of selling identical items using auctions and a posted price simultaneously. We develop a detailed model of consumer behavior and show how to calculate the seller's expected revenue from the dual channel for a stochastic number of bidders. We describe the characteristics of an optimal design of the dual channel through numerical examples in §3.3 and we conclude in §3.4. The proofs for propositions in this chapter are in §3.5.

3.2 The Model

We model an online seller who offers identical items using two selling mechanisms, posted price and auctions, simultaneously. The auctions have a fixed duration and are then repeated. The seller's objective is to maximize his revenue per unit time. The seller chooses the auction duration T, the quantity to auction q, and the posted price p. Without loss of generality, we assume that the marginal cost of each unit is zero (if this is not so, consumers' valuations of the product can be taken net of the marginal cost). The seller's publicly declared reserve price is R.[1] We also assume that the seller can satisfy any demand. Consumers visit the web site according to a Poisson process with rate λ, and

[1] A public reserve price is equivalent to setting a minimum initial bid.

each consumer is interested in purchasing one unit of the good.[2] Consumers have independent private values for the good. We assume that each consumer's valuation, V, is independently drawn from a probability distribution with cumulative density function $F(\,)$ with support set $[\underline{v}, \bar{v}]$, where $\underline{v} \geq R$.

Since the two selling channels are being offered simultaneously on the same platform or in the same space (the Internet), we assume that consumers can observe both channels on arrival, with no additional costs. Hence, consumers are fully informed: they observe the item's posted price, whether an auction is currently offered, the auctioned quantity and the time remaining in the auction. They do not know the number of other bidders.

We model the auctions using the sealed bid *(q+1)- price* format with risk-neutral bidders having unit demand and independent private values for the good. In a sealed bid *(q+1)- price* auction, the winners are the bidders with the q highest bids (q being the auctioned quantity), and each pays a price equal to the $(q+1)$ highest bid (the highest losing bid). In such a sealed bid *(q+1)- price* auction, with no posted price offering, the dominant strategy for each bidder is to bid his true valuation of the item (Milgrom, 1987). By doing so, the bidder is setting an upper bound on the price that he is willing to pay: he will accept any price below his reservation price and none above.

Most online auctions are conducted using the open, ascending bid English auction format, where bidders can observe the lowest bid needed to win at every moment of the auction and, on some web sites, can see how many other bidders there are. This suggests that consumers have more information available to them when choosing between the posted price and auction participation than in the sealed bid auction we model. In practice, however, last minute bidding or "sniping" is very prevalent (Roth and Ockenfels 2002). This suggests that actually very little information is available to the consumers, since they do not know how many other bidders are lurking in the background nor do they have any indication of what these other bidders are willing to bid. The result is a de facto sealed-bid auction.

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[2] We assume a constant arrival rate. Notice that in B2C auctions, although there is a lot of last minute activity because of "sniping," there is a distinction between arrival times and activity/bidding times.

In our setting, the existence of the posted price option adds considerable complexity to the analysis of the auction because it splits the consumers into several subgroups (see Figure 3.1).

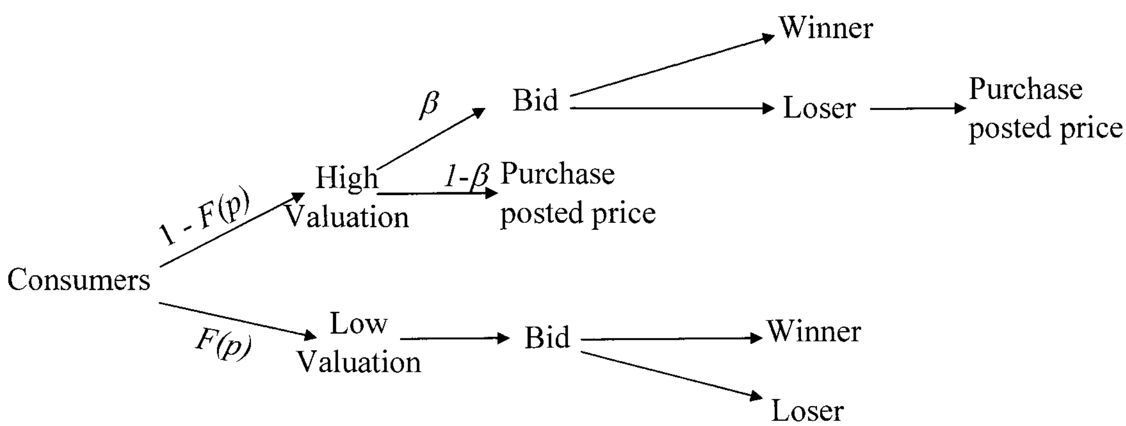

Figure 3.1: Schematic of customer splitting in the presence of dual channel.

The posted price first splits the consumers into those with low valuations (i.e., valuations less than the posted price), and those with high valuations (i.e. valuation greater than or equal to the posted price). In this model, all low valuation consumers become bidders in the auctions; we explain the rationale for this in §3.2.1. A fraction β of the high valuation consumers also become bidders while a fraction 1-β purchase at the posted price. The probability of participation, β, depends on the design variables (q, T, and p) and much of the analysis in this section is devoted to its determination. Some of the bidders win the auction and some lose. The high valuation bidders who lose will purchase at the posted price at the conclusion of the auction. To explain the choices made by consumers depicted in Figure 3.1, we next describe our model of consumer behavior. Table 3.1 summarizes the notation used throughout the Chapter.

Decision variables:

p	posted price.
q	auctioned quantity (per auction).
T	auction length.
Model parameters:	
λ	arrival rate of consumers at the web site.
$F(v)$	cumulative density function of consumers' valuation distribution with support $[\underline{v}, \bar{v}]$.
w	delay cost incurred by high valuation consumers per unit time.
R	seller's public reserve price.
Other notation:	
p_a	auction closing price.
N^+	random number of bidders who value the item for more than its posted price.
N^-	random number of bidders who value the item for less than its posted price.
$\bar{t}$	time remaining in an auction beyond which high valuation consumers will not participate for the (symmetric) participation-strategy equilibrium.
$O\{x,y\}$	expected value of the x^{th} order statistic of y draws from the consumer valuation distribution truncated on $[\underline{v}, p]$.
β	probability a high valuation consumer participates in the auction.

Table 3.1: Summary of notation for chapter 3

3.2.1 Consumer Behavior

As noted in the literature review, most auction models examine markets where auctions are the sole selling mechanism and the number of bidders is deterministic. In such markets consumers face a simple choice between bidding and not. In the absence of auction-related costs, the expected value from bidding is always nonnegative, so the set of bidders is the same as the set of arrivals: each arriving consumer chooses to participate in the auction rather than stay out of the market. Here, we model the behavior of risk-neutral consumers when the seller offers both auctions and a posted price. We examine how consumers choose between the available channels and define a weakly dominant bidding strategy (and thus a dominant equilibrium) for those who choose to bid.

When a risk-neutral consumer has the option of a posted price channel, he bases his choice on his expectation of a greater surplus. When consumers choose the auction channel, it is because they believe that there is an opportunity to purchase the good at a discount over the posted price. Yet there are costs to participating in an auction, so in expectation the auction price discount must exceed these costs. There are essentially two auction participation costs: the cost of monitoring and making bids and the cost of deferring the purchase of the good until the end of the auction. There is empirical evidence that these costs influence the behavior of auction participants.

Hann and Terwiesch (2003) study bidder behavior on a German name-your-price service that allows bidders to update their bids after only a few minutes. They find that bidders do not bid very frequently with small increments, as one would expect, but rather bid only a few times (typically less than four) with significant bid increments. They explain this behavior as demonstrating that there is a significant participation cost in these online negotiation settings. Lucking-Reiley (1999) conducts experiments with Internet auctions of collectible trading cards, comparing the profit generation of Dutch auctions with that of first-price auctions, which theory predicts are equivalent. He finds, to the contrary, that Dutch auctions had closing prices on average 30 percent higher than first-price auctions. He speculates that one possible reason is the fact that the Dutch auctions were much longer and bidders might have been impatient to complete their purchase. Motivated by Lucking-Reiley's observation, Carare and Rothkopf (2001) develop decision and game theoretic models of slow Dutch auctions to show how including auction transaction costs related to the auction duration alter their outcomes. In their models the value the bidder receives from the auctioned good decreases with the time spent in the auction; i.e., there is a delay cost. We use a similar approach in our model.

Some auction mechanisms require more active participation by the bidders and therefore have higher participation costs. The sealed-bid auction we model does not require the bidders to constantly monitor the auction's progress or to analyze the behavior of other bidders. In our setting, therefore, it is reasonable to assume that the most significant component of the auction participation cost is the delay cost.

To develop the intuition behind our model we consider the following numerical example. **Example 1:** A seller is offering two computer keyboards in an auction while simultaneously selling them online for \$100. Let us say seven consumers $\{B_1, B_2, B_3, B_4, B_5, B_6, B_7\}$ with respective valuations {5, 10, 80, 90, 101, 110, 120} arrive at the website. Consumers B_1, B_2, B_3, and B_4 have no other option but to bid in the auction because the posted price is too high for them. We demonstrate in Lemma 1 that their optimal bidding strategy is to bid their valuations. If only these four consumers bid, the closing price of the auction will be \$10, with B_3 and B_4 winning. What about B_5? He can purchase the keyboard at the posted price and have it shipped to him immediately while receiving a surplus of \$1, but he might be able to do better by bidding in the auction. We demonstrate in Lemma 1 that B_5's optimal bidding strategy is to bid the posted price \$100. If he were to bid this way against consumers B_1, B_2, B_3 and B_4, he would win and pay \$80, yielding a surplus of \$21. If B_5 arrived at the auction 3 days before it ended, choosing to participate would force him to incur a waiting cost because if he wins he will receive the good three days later than if he had purchased at the posted price. If this cost is \$3 per day his surplus would be reduced to \$12. If consumer B_6 also bid in the auction, the closing price would increase to \$90, reducing B_5's surplus further to just \$2. We can see from this example that a high valuation consumer may find it worthwhile to participate in the auction if he anticipates receiving a discount over the posted price larger than the delay cost, where the discount is determined by the number and types of the other bidders.

3.2.1.1. The Consumer's Problem

Low valuation consumers, those with $V < p$, cannot buy the item for its posted price because the value they get from doing so is negative, so they choose between bidding and staying out of the market. These consumers prefer to receive the item earlier rather than later, and they may choose not to bid if the remaining time of the auction is significantly long (hence, we later set an upper bound on the feasible auction length when solving the seller's optimization problem in §3.3). However, since they have no other option for obtaining the item, we assume that the delay cost per unit time perceived by these consumers is significantly lower than the delay cost per unit time perceived by consumers who can obtain the item instantly by paying the posted price. To simplify the problem, we therefore assume that the delay cost per unit time is $w = 0$ for consumers with $V < p$. The optimization problem faced by these consumers is

$Max\{U_-^A(V), 0\}$, where

$$U_-^A(V) = Max_{b\in[0,\infty)} \Pr(win|b)V - E[auction\_payment \mid b]. \tag{3.1}$$

We define $U_{-}^{A}(V)$ as the maximum expected value from participating in the auction for a consumer with valuation $V<p$, where the expected value is taken over the bids of all other bidders in the auction. Pr(*win*|*b*) is the probability that the consumer wins the item in the auction by bidding *b*, and E[*auction_payment*|*b*] is the expected auction payment by a bidder who bids *b*. Notice that E[*auction_payment*|*b*] differs from the expected auction price when bidding *b*, E[p_a|*b*], because when the consumer loses in the auction his expected auction payment is zero, but the auction price is not.

High valuation consumers, those with $V \geq p$, would buy the item for its posted price if auctions were not offered. High valuation consumers choose between buying the item for its posted price and participating in the auction. It is never optimal for these consumers to do nothing, because their utility from buying the item for the posted price is nonnegative. We assume that when high valuation consumers purchase the item for its posted price they obtain the item instantly. When they choose to bid, they are choosing to experience a delay in obtaining and using the item, because they must wait until the end of the auction. Hence, when choosing to bid, these consumers incur a delay cost that is an increasing function of the time remaining until the end of the auction. $U_{+}^{A}(V)$ denotes the maximum expected value from participating in the auction, for a consumer with valuation $V \geq p$, and we define $U_{+}^{B}(V)$ as the value a consumer with valuation V derives from purchasing the item for the fixed price. A high valuation consumer, arriving with t^{e} time units remaining in the auction, solves the following optimization problem:

$$Max_{i \in A,B} U_{+}^{i}(V),$$

where

$$U_{+}^{A}(V) = Max_{b \in [0,\infty)} \Pr(win|b)V - E[auction\_payment \mid b] + \Pr(lose|b)(V-p) - wt^{e} \quad (3.2)$$

$$U_{+}^{B}(V) = V - p$$

Pr(*win*|*b*) is the probability that the consumer wins the item in the auction by bidding *b*, and Pr(*lose*|*b*) = 1- Pr(*win*|*b*). The consumer evaluates the expected payoff from bidding, using an optimal bidding strategy, and compares it with the payoff from purchasing the item for the posted price. The first two terms of the RHS of (3.2) give the expected value from bidding *b* when the product is not offered for a posted price. The existence of a posted price offering has two opposing effects on the auction's value for a high valuation consumer:

- $\Pr(lose|b)(V-p)$: If the customer loses the auction, we assume he can and will purchase the item for the same posted price, with a payoff of (*V-p*). Hence, the existence of a posted price increases the expected value from participating in the auction by reducing the cost of losing the auction. We acknowledge that this assumption might not capture the bidder's optimal behavior, and so we might be underestimating the high valuation consumer's payoff when choosing to bid. However, we do not believe that this assumption will be restrictive in practice. A high valuation consumer, who has entered one auction and lost, faced a lot of competition from other high valuation consumers and thus has no reason to expect a different outcome in a future auction; hence, he will be less likely to bid again.
- $-wt^e$: Since the consumer could have bought the product for the posted price and obtained the item instantly, he incurs a delay cost when he chooses to bid and wait until the end of the auction to receive the item. We assume that this delay cost is linear in the time remaining until the auction ends. We recognize that in some cases the delay cost is not linear in the waiting time. For example, if consumers must have the item by a given date (as a birthday gift or for a scheduled trip) the delay cost is zero up to that date but infinite afterwards. However, since different consumers would have different step cost-functions (they need the item by different dates) the expected delay cost of a consumer would depend on the distribution of step functions. Exploring the effect of different assumptions regarding the functional form of the delay cost on the optimal consumer behavior can be done in future research. In addition, it may be that *w,* the delay cost per unit time, is an increasing function of *V*. That is, a consumer who values the item more also relates a higher cost to a delay in using the item. To simplify the following analysis, we assume that *w* is positive and independent of *V* for consumers with $V \geq p$.

3.2.1.2. Optimal Auction Participation and Bidding

In determining the consumers' participation and bidding strategies, we restrict our attention to symmetric equilibria, in which all consumers adopt the same strategy. This is reasonable since consumers are symmetric in the sense that their valuations and arrival times are drawn from the same distributions. We determine the strategy of a high valuation consumer in two steps. In Lemma 3.1 we first derive a weakly dominant bidding strategy for all consumers who have chosen to participate in the auction. In Proposition 3.1, we then use this strategy as an input in determining a unique symmetric equilibrium for a high valuation consumer's participation strategy.

We model the sealed bid auction as a static game of incomplete information. Conditioned on having chosen to participate in the auction, the arrival time of the consumer is irrelevant to the bidding strategy, because the delay cot is sunk. The action space for bidder i is the space of possible bids, $B_i=[0,\infty)$, and the type space is $T_i = [\underline{v}, \bar{v}]$, the support of the consumer valuation distribution. A strategy, $b(V)$, is a mapping from the type space to the action space. Because valuations are independent, player i believes that V_k for every $k \neq i$ is drawn from the CDF $F(\,)$.

Next, we find a bidding strategy, $b_i(V_i)$, such that for any given number of bidders and combination of other bidders' actions $b_{-i} = (b_1, b_2, \ldots, b_{i-1}, b_{i+1}, \ldots b_{N^+ + N^-})$, and for any other bidding strategy, $b_i'(V_i)$, $U_i(b_i(V_i), b_{-i}, V_i) \geq U_i(b_i'(V_i), b_{-i}, V_i)$ with strict inequality for some b_{-i}. That is, we find a weakly dominant strategy for this static game of incomplete information, and such a strategy provides a dominant equilibrium.

Lemma 3.1: *A weakly dominant bidding strategy for risk-neutral bidders with independent private values in a sealed bid (q+1)- price auction that is conducted parallel to a posted price, p, is the following:*

$$b(V) = \begin{cases} V & \text{for } V < p \\ p & \text{for } V \geq p \end{cases} . \tag{3.3}$$

All proofs are at the end of the Chapter, in Section 3.5.

Note that the existence of an outside option at price p puts an upper bound of p on the bids placed by high valuation consumers. This is in contrast to the optimal bidding strategy in a traditional sealed-bid second-price auction, in which it is optimal to bid one's valuation. Losing the auction in our setting is less of a loss, because of the infinite posted price supply.

For consumers with $V < p$, the value from participating in the auction is non-negative, so all of these consumers choose to bid rather than stay out of the market. Hence, the number of these participants in the auction is the same as the number of arrivals. The number of low-valuation participants, N^-, is a random variable from a Poisson distribution with rate $\lambda_l = \lambda F(p)$. Consumers with $V \geq p$ choose to participate in the auction rather than to buy the item for its posted price if and only if

$$\Pr(win|p)V - E[auction\_payment \mid p] + \Pr(lose|p)(V - p) - wt^e \geq V - p. \qquad (3.4)$$

Proposition 3.1: *In a dual channel with a sealed bid (q+1)- price auction in which bidders follow the strategy of Lemma 3.1, high valuation consumers participate in the auction iff*

$$D \equiv \Pr(win|p)p - E[auction\_payment \mid p] \geq wt^e, \qquad (3.5)$$

and there exists a unique (symmetric) equilibrium, in which all high valuation consumers choose to bid if and only if $t^e \leq \bar{t}$, *where* $0<\bar{t}<T$ *and is given by the solution of the fixed point equation*

$$D(\bar{t}) = w\bar{t} \qquad (3.6)$$

if $D(T) < wT$, *and* $\bar{t}=T$ *if* $D(T) \geq wT$.

The proof is in Section 3.5.

We note that $\Pr(win|p)p - E[auction\_payment \mid p]$ is the expected discount a high valuation consumer gets over the posted price if he participates in an auction. We use $D(\bar{t})$ to denote the expected discount a high valuation consumer gets over the posted price when all other high valuation consumers use the threshold $\bar{t}$ to choose whether to participate in the auction or to buy the item for the posted price.

Based on Proposition 3.1, we conclude that under fairly general conditions high valuation consumers use a threshold policy to choose between buying the item for its posted price and bidding in the auction. If the remaining time of the auction observed by a high valuation consumer upon his arrival exceeds the threshold, the consumer chooses to purchase the item for its posted price. If the remaining time of the auction is less than the threshold, $\bar{t}$, the consumer chooses to participate in the auction. This result is shown in Figure 3.2. The numbered dots in Figure 3.2 depict the arrival times and valuations of bidders. The horizontal axis represents the time remaining in the auction when the consumer arrives, and the vertical axis represents the consumer's valuation of the good being sold.

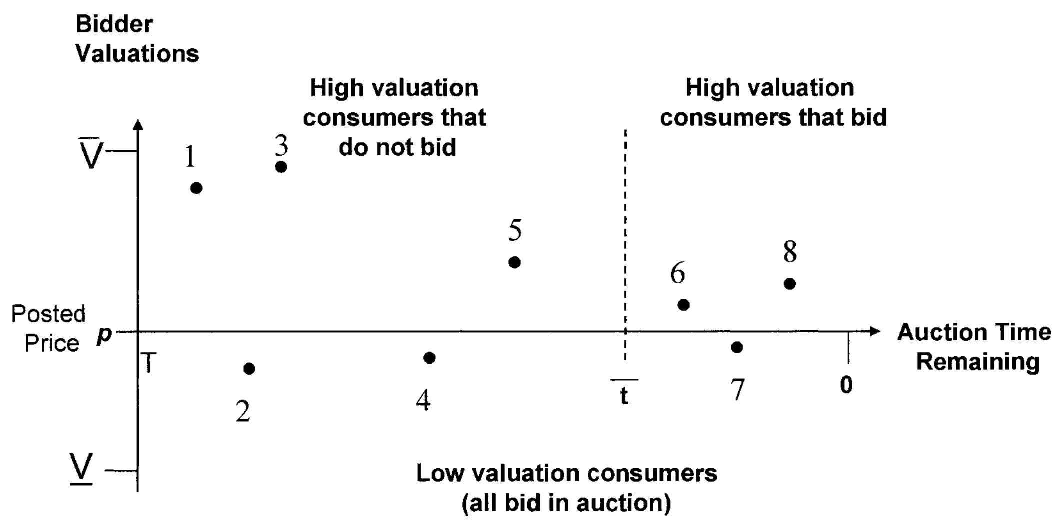

Figure 3.2: The dynamics of auction participation
(p – posted price, T – auction duration, $\bar{t}$ - participation threshold, $[\underline{v}, \bar{v}]$ - valuation support)

Figure 3.2 depicts the two types of consumer segmentation occurring in the dual channel. The posted price splits the consumers into low and high valuation groups. The low valuation consumers all bid in the auction regardless of their arrival times (consumers 2, 4, and 7). The threshold time $\bar{t}$ segments the high valuation consumers into those who bid and those who do not. Consumers 1, 3, and 5 all arrive with more than $\bar{t}$ time units remaining in the auction and therefore have delay costs that exceed their expected discount from participating in the auction. On the other hand, consumers 6 and 8 arrive late enough that it is worthwhile for them to bid in the auction and delay their purchase of the item.

Since Poisson arrivals are uniformly distributed over a fixed time interval, the fraction of high valuation consumers that participate in the auction is given by $\beta = \bar{t}/T$, where $\bar{t}/T$ is the probability that a consumer arrives in the last $\bar{t}$ units of time of the auction. We conclude that the number of high valuation consumers who bid, N^+, has a Poisson distribution with rate $\lambda_2 = \lambda(\bar{t}/T)(1 - F(p))$ and that the smaller the value of $\bar{t}$, relative to T, the more effectively the seller has segmented the low and high valuation consumers (i.e., fewer high valuation consumers are expected to participate in the auction).

It is important to note that the results in Proposition 3.1 depend on the consumers' assumption that the seller's capacity is unlimited. When the capacity is limited, the probability of being able to purchase the item for the posted price at the end of the auction is less than one because of the positive probability that the seller will run out of stock. The participation threshold would then be dependent upon the consumer's valuation, V. Hence, our suggested model holds when consumers believe that the probability that the seller will run out of stock during the auction is zero. This is plausible when the auction's length is relatively short and the seller's capacity is assumed to be large. A different model is needed for items such as airline tickets, end-of-season items, or refurbished goods, for which the probability of being out of stock is significant.

3.2.2 The seller's optimization

Recall that N^- and N^+ are defined, respectively, as the number of low and high valuation consumers who bid. We have shown above that N^- and N^+ are random variables from Poisson distributions with rates λ_1 and λ_2 respectively, where $\lambda_1 = \lambda F(p)$ and $\lambda_2 = \lambda(1-F(p))\bar{t}/T$. The seller determines these rates by selecting T, q, and p. The seller's decision problem can thus be formulated as follows:

$$\underset{T,q,p}{Max}\ \frac{1}{T}\left[E[\pi_a]+\left(\lambda T(1-F(p))-E[N^+]\right)p+p\sum_{x=q+1}^{\infty}(x-q)\frac{e^{-\lambda_2 T}(\lambda_2 T)^x}{x!}\right], \qquad (3.7)$$

where

$$E[\pi_a]=qp\sum_{x=q+1}^{\infty}\frac{e^{-\lambda_2 T}(\lambda_2 T)^x}{x!}+q\sum_{x=0}^{q}\sum_{y=q+1-x}^{\infty}O\{q-x+1,y\}\frac{e^{-\lambda_1 T}(\lambda_1 T)^y}{y!}\frac{e^{-\lambda_2 T}(\lambda_2 T)^x}{x!}+$$
$$R\sum_{x=0}^{q}\sum_{y=0}^{q-x}(x+y)\frac{e^{-(\lambda_2+\lambda_1)T}(\lambda_2 T)^x(\lambda_1 T)^y}{x!\,y!}$$

The first term of Equation 3.7 is the expected revenue from the auction. The second term is the expected revenue from sales for the posted price during the auction. The number of purchasers for the posted price equals the number of high valuation consumers

(consumers with $V \geq p$) who arrive during the auction less those who choose to bid. The last term in Equation 3.7 is the expected revenue from sales to high valuation consumers who lost in the auction.

If the number of bidders is less than q, the auction price is the seller's reserve price R. Note that, in our model, R is a parameter, not a decision variable, and only consumers with valuations $V \geq R$ are relevant to the analysis. Our model could be used as an engine to identify the optimal reserve price in situations in which the reserve price is public knowledge. Some online auctions such as those conducted on eBay, SamsClub.com, and Compusaauctions.com, allow sellers to post secret reserve prices. A secret reserve price will deter some bidders from participating in the auction as Bajari and Hortacsu (2003) observe. Our model is not designed to capture this effect.

3.3 Design of the dual channel

To develop our intuition of the seller's perspective, consider a numerical example modeled on Example 1. **Example 2**: Seven consumers $\{B_1, B_2, B_3, B_4, B_5, B_6, B_7\}$ with respective valuations {5, 10, 80, 90, 101, 110, 120} come to a website selling computer keyboards. If there were only a posted price channel with price p = \$100, the seller would only sell to B_5, B_6, and B_7, for a revenue of \$100 + \$100 + \$100 = \$300. If he offers the two-unit auction and none of the high valuation bidders participate, he will earn an additional \$20 (for a total of \$320), because B_4 and B_3 will win at the price of B_2's bid of \$10. If B_5 and B_6 choose to bid, they will win at a price of \$90 and the seller's total revenue will be \$100+ \$90 + \$90 = \$280, a decrease of \$20 with no additional sales. If, however, only B_6, among the high valuation bidders, participates in the auction, then B_6 and B_4 will win the auction at a closing price of \$80, leading to \$200 of posted price revenue (from B_5 and B_7) and \$160 of auction revenue (from B_6 and B_4), for a total of \$360.

This example illustrates that when high valuation consumers do participate in the auction there is the side benefit of an increase in the auction price. Hence, although the design of the auction should aim to discourage high valuation consumers from bidding, which is equivalent to narrowing the time period in which high valuation consumers choose to bid (reducing $\bar{t}$), as depicted in Figure 3.2, this objective is tempered by the positive effect high valuation bidders have on auction prices. The seller also has two other related goals: to increase the number of units sold in the auction per unit time, and to increase the prices paid in the auction. To identify auction design strategies, we consider how the choice of q and T affects these three goals. We summarize this reasoning in Table 3.2.

Goal:	Reduce $\bar{t}$	Increase auction price	Increase auction sales per unit of time
Strategy 1:	Large T	Large T	Large q
Strategy 2:	Small q	Small q	Small T

Table 3.2: Potential auction design strategies for high arrival rates

Increasing T will increase the auction price, because it increases the number of bidders who cannot buy at the posted price; i.e., it increases competition for the auctioned items (defined as the number of bidders per unit auctioned). A higher auction price means that $\bar{t}$ will be smaller, because high valuation consumers need a smaller delay cost to make the auction worthwhile. On the other hand long auctions decrease the total auction sales per unit time, because it takes longer to sell every unit auctioned. Short auctions have the opposite effect on each goal. Decreasing q is another way to increase competition in the auctions. Small lot sizes will increase auction prices and as a result drive away high valuation consumers, while at the same time few items are sold via auction, and the auction sales are smaller. Large lot sizes can compensate for the negative aspects of long auctions, and short auctions can compensate for the negative aspects of small lot sizes. The two strategies described in Table 3.2 follow naturally. The seller should either: (1) set one-unit auctions with length decreasing in the consumer arrival rate, or (2) set long auctions with lot size increasing with the consumer arrival rate. When the arrival rate is low, the seller's main concern is the auction price and cannibalization of sales at the posted price. In this case, the seller may need to set maximum length, one-unit auctions in order to sell auctioned units above marginal cost, or it may even become suboptimal to add the auctions.

3.3.1 Numerical experiments

In the following numerical experiments, we assume that consumer valuations follow a uniform distribution over [$0, $100]. Hence, we can use a closed-form expression for the expected value of the order statistics, $O\{x, y\}$, in our model. We use Equation 3.7 to calculate the seller's expected revenue for given λ, w, q, T, p, and $\bar{t}$. We vary the arrival rate λ between 0.5 and 60 consumers per day and vary the delay cost, w, between $0.1 and $4 per day. For each combination of λ and w, we determine the optimal values of q, T and p.

Based on Proposition 3.1, the equilibrium auction participation strategy for high valuation consumers consists of a threshold value, $\bar{t}$, given by the solution of the fixed point equation $D(\bar{t}) = w\bar{t}$, where $D(\bar{t})$ is the expected auction discount over the posted price for a high valuation bidder, assuming the number of additional high valuation bidders and the number of low valuation bidders follow the relevant Poisson distributions. However, to assume that consumers can actually solve $D(\bar{t}) = w\bar{t}$ for $\bar{t}$ means that we assume consumers are familiar with Poisson distributions, the properties of Poisson processes, and the manipulation of infinite sums (see Appendix for the explicit expression of $D(\bar{t})$, when using Poisson distributions for the number of bidders from each group). Although the seller is likely to have software tools that can aid in collecting data on consumer valuations and arrival patterns in order to determine the optimal values of the design parameters using Poisson distributions (i.e., using Equation 3.7), it is reasonable to assume that the average consumer does not have such capabilities. In practice, it is most reasonable to expect that consumers will use some heuristic to determine their expected discount from the auction and therefore their participation threshold. Because we do not know what heuristic individual bidders may be using, we propose a single heuristic to represent their behavior. We assume that consumers use expected (average) values rather than distributions of number of bidders. In other words, when all other high valuation consumers use a threshold $\bar{t}_h$, a high valuation consumer estimates his expected auction discount assuming the number of low valuation bidders is $\lambda \Pr(V < p)T$ and the number of additional high valuation bidders is $\lambda \Pr(V \geq p)\bar{t}_h$.

Since auction prices must be calculated using integer numbers of bidders, and $\lambda \Pr(V < p)T$ and $\lambda \Pr(V \geq p)\bar{t}_h$ may be non-integer, we further assume that the high valuation consumer interpolates between the nearest integer values for the expected number of low and high valuation bidders to determine the expected auction discount. Specifically, we have:

Heuristic 3.1: A high valuation consumer evaluates his expected auction discount over the posted price as if the number of (other) high valuation bidders (bidders with $V \geq p$) arriving in a period of length t is $\lceil \lambda \Pr(V \geq p)t \rceil$ with probability ρ and $\lfloor \lambda \Pr(V \geq p)t \rfloor$[3] with probability (1- ρ), where $\rho = \lambda \Pr(V \geq p)t - \lfloor \lambda \Pr(V \geq p)t \rfloor$, and the number of low valuation bidders arriving in a period of length t is $\lceil \lambda \Pr(V < p)t \rceil$ with probability γ and $\lfloor \lambda \Pr(V < p)t \rfloor$ with probability (1- γ), where $\gamma = \lambda \Pr(V < p)t - \lfloor \lambda \Pr(V < p)t \rfloor$.

We do not claim that any bidders actually make the calculations described in the heuristic. However, we think that this heuristic captures the interactions among all the information potentially available to a bidder, except the nature of the stochastic process governing bidder arrivals, and therefore can be viewed as representative of what a bidder's heuristic might result in. We use $D_h(\bar{t})$ to denote the expected auction discount over the posted price for a high valuation bidder using Heuristic 1 and assuming all other high valuation consumers use the threshold $\bar{t}$. That is

$$\begin{aligned} D_h(\bar{t}) = {} & \rho(\bar{t})\gamma\, d\left(\left\lceil \lambda \Pr(V \geq p)\bar{t} + 1 \right\rceil, \left\lceil \lambda \Pr(V < p)T \right\rceil \right) + \\ & (1-\rho(\bar{t}))(1-\gamma)\, d\left(\left\lfloor \lambda \Pr(V \geq p)\bar{t} + 1 \right\rfloor, \left\lfloor \lambda \Pr(V < p)T \right\rfloor \right) \\ & (1-\rho(\bar{t}))\gamma\, d\left(\left\lfloor \lambda \Pr(V \geq p)\bar{t} + 1 \right\rfloor \left\lceil \lambda \Pr(V < p)T \right\rceil \right) + \\ & \rho(\bar{t})(1-\gamma)\, d\left(\left\lceil \lambda \Pr(V \geq p)\bar{t} + 1 \right\rceil \left\lfloor \lambda \Pr(V < p)T \right\rfloor \right) \end{aligned} \tag{3.8}$$

where $d(x,y) = \begin{cases} p - O\{q - x + 1, y\} & \text{if } x \leq q \text{ and } x + y > q \\ 0 & \text{if } x > q \\ p - R & \text{if } x + y \leq q \end{cases}$.

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[3] $\lceil\ \rceil$ gives the closest larger integer, and $\lfloor\ \rfloor$ gives the closest smaller integer.

Proposition 3.2: *When consumers use Heuristic 3.1, a unique (symmetric) equilibrium strategy for high valuation consumers is given by the solution of the fixed point equation* $D_h(\bar{t}) = w\bar{t}$, *if* $D_h(T)<wT$, *and by* $\bar{t}=T$ *otherwise,*

All proofs are in Section 3.5.

To summarize, given (w, λ) we use Equation 3.7 to calculate the seller's expected revenue, when consumers use Heuristic 3.1, for each triplet (q, T, p) to find the optimal dual channel design. In the Appendix, we present the results of numerical experiments analyzing performance when consumers do not use a heuristic but instead calculate the expected auction discount exactly, to determine the participation threshold $\bar{t}$.

Recall that in our model consumers with $V < p$ always participate in the auction because they have no other purchase options. Yet, it is reasonable to assume that even such bidders will not join an auction if its duration is too long. In other words, we assume that there is a maximum auction duration H, such that consumers ignore the auction until the time remaining in the auction is less than or equal to H. For the base case we assume that $H = 168$ hours (seven days). Experiments with different values of H did not qualitatively change our results (see Appendix). To reduce the computational burden we treat T as a discrete variable $T \in \{1 \text{ hour}, 2 \text{ hours}, \ldots, H \text{ hours}\}$. Table 3.3 summarizes our findings for $H = 7$ days.

λ : w:	**\$0.1/day**	**\$0.5/day**	**\$1/day**	**\$2/day**	**\$3/day**	**\$4/day**
0.5/day	No auction, P=\$50					1;H; \$56
1/day	1; H; \$52	1; H; \$53	1; H; \$53	1; H; \$53	1; H; \$54	1; H; \$54
2/day	1; 134; \$52	1; 135; \$52	2; H; \$53	2; H; \$53	2; H; \$54	3; H; \$56
5/day	1; 53; \$52	1; 55; \$52	6; H; \$52	7; H; \$54	8; H; \$55	8; H; \$57
10/day	1; 27; \$52	1; 27; \$52	13; H; \$52	16; H; \$54	18; H; \$56	19; H; \$58
20/day	1; 13; \$52	1; 14; \$52	30; H; \$52	35; H; \$54	38; H; \$56	39; H; \$58
30/day	1; 9; \$52	1; 9; \$52	47; H; \$52	54; H; \$54	58; H; \$57	60; H; \$58
40/day	1; 7; \$52	1; 7; \$52	1; 7; \$52	75; H; \$55	78; H; \$57	80; H; \$58
50/day	1; 5; \$52	1; 5; \$52	1; 5; \$52	94; H; \$55	98; H; \$57	100; H; \$58
60/day	1; 4; \$52	1; 4; \$52	1; 4; \$52	113; H; \$55	118; H; \$57	121; H; \$58

Table 3.3: The optimal design (q, T [Hr], p[\$]) for various values of consumer arrival rate, λ, and delay cost, w, when H=168 hours and V is uniformly distributed on [\$0, \$100].

Reinforcing our previous discussion, we see that the optimization results in one of the two cases we predicted, one-unit auctions with the length of the auction decreasing in the arrival rate or long auctions (seven days, the maximum length) with the size of the auction lot increasing in the arrival rate. Which of the two settings is optimal depends on the delay cost per unit time incurred by high valuation consumers and on the consumers' arrival rate.

When w is low, the only way to deter high valuation consumers from bidding is to reduce the size of the auction lot. A long auction will not work, because if w is sufficiently low, high valuation consumers will always choose to bid (regardless of the time they arrive during the auction). For small values of w, the seller should therefore offer one-unit auctions, and the length of the auctions should increase as the arrival rate decreases. When the delay cost, w, is high, it deters high valuation consumers from bidding. The seller should set the auction length to the maximum and increase the size of the auction lot as the consumers' arrival rate increases. When arrival rates are low, the main concern is the auction price and cannibalization of the posted price channel, so it becomes optimal to offer one-unit auction with the maximum length of 7 days (which is an extreme case of each of the above two strategies). As the arrival rate decreases, the single channel (only posted price) might outperform the dual channel. As w increases, the dual channel outperforms the single channel even for smaller arrival rates.

We can also see from Table 3.3 that the posted price is increasing as w increases and as λ increases, when auctions are long (H hours). When there is no auction, increasing the posted price from the optimal point leads to a loss of sales from those consumers who are priced out that cancels any revenue gains from higher prices. When there is an auction, the consumer who was priced out by a price increase will not be completely lost, because he may purchase in the auction, and his participation increases the auction price. As the website traffic, λ, increases it becomes more attractive to have more sales via auctions. Increasing the posted price means that a larger proportion of the consumers are targeted by the auction channel. In Table 3.4, we see that as λ increases the fraction of revenues coming from auctions increases.

λ: w:	$0.1/day	$0.5/day	$1/day	$2/day	$3/day	$4/day
1/day	$36.97; 21%	$37.24; 21%	$36.5; 20%	$35.26; 19%	$35.17; 19%	$34.37; 19%
2/day	$42.11; 15%	$41.85; 14%	$43.7; 24%	$38.18; 23%	$34.43; 22%	$47.67; 32%
5/day	$42.05; 15%	$42.26; 14%	$45.98; 30%	$45.81; 35%	$44.38; 38%	$44.75; 38%
10/day	$42.25; 15%	$42.17; 15%	$47.53; 34%	$46.89; 41%	$46.05; 44%	$45.54; 45%
20/day	$41.87; 15%	$42.52; 14%	$48.17; 40%	$47.32; 45%	$46.43; 47%	$46.06; 47%
30/day	$42.21; 15%	$42.21; 15%	$48.43; 42%	$47.46; 46%	$47.41; 48%	$46.04; 48%
40/day	$42.52; 14%	$42.52; 14%	$42.52; 14%	$48.25; 49%	$47.46; 49%	$46.17; 48%
50/day	$41.42; 15%	$41.42; 15%	$41.42; 15%	$48.31; 49%	$47.5; 49%	$46.25; 48%
60/day	$40.99; 16%	$40.99; 16%	$40.99; 16%	$48.36; 49%	$47.52; 49%	$46.21; 48%

Table 3.4: Expected auction price and fraction of total revenue coming from auction channel, (Auction price; Auction revenue fraction), for various values of consumer arrival rate, λ, and delay cost, *w*.

Figure 3.3 shows how the posted price and the expected auction price change with the delay cost, *w*, when arrival rate is 20 per day. In the range of *w* values where it is optimal to have one unit auctions, the posted price is higher than the optimal price in the absence of auctions, and the expected auction price is non-decreasing in *w*. In the range of *w* values where it is optimal to have long auctions with multiple units of the item, the posted price is increasing in *w*, and the expected auction price is decreasing in *w*. This shows that as *w* increases, the two channels become more "separated"; that is, the gap between the posted price and the expected auction price increases. Hence, as *w* increases both the delay cost and the expected auction discount increase.

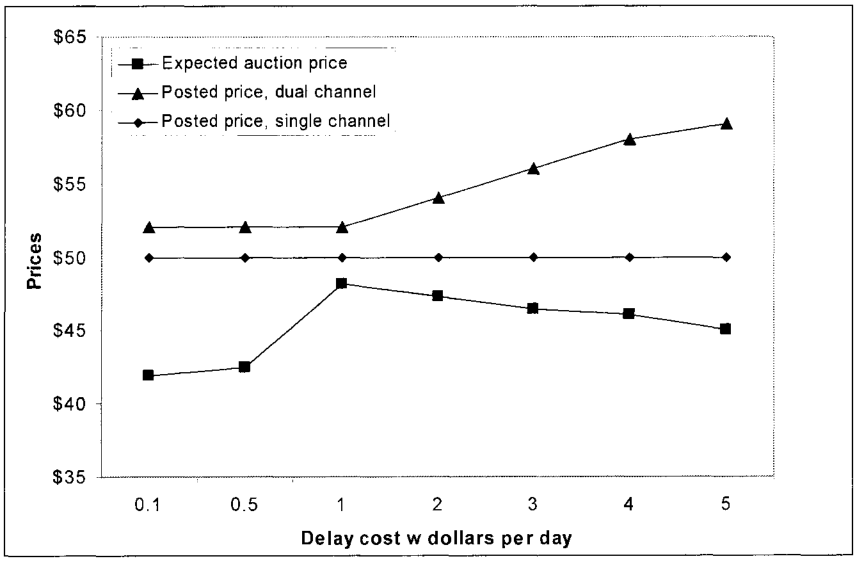

Figure 3.3: The optimal posted price and the expected auction price as a function of the delay cost, *w*, for λ=20/day.

When w gets large enough, the seller can offer more units in the auctions without there being a complete collapse of the posted price channel. The result is that the reduction in posted price sales (due to the higher posted price) is more than made up for by the increase in sales in the auction (due to the larger auction lot size).

In Table 3.5 we show how the units sold are allocated among the different groups of consumers: high valuation auction participants, low valuation auction participants, and high valuation consumers who purchase at the posted price. When it is optimal to have short one-unit auctions, the number of units bought by high valuation consumers in the auctions is small relative to the number they buy at the posted price. When it is optimal to have long auctions with larger lot sizes, the auctions become responsible for a significant fraction of the sales to high valuation consumers. We also see that more auction sales tend to go to the high valuation consumers than to the low valuation consumers, and that the fraction of high valuation consumers who buy the item via the auctions (given by the expected number of units bought by high valuation consumers via auctions divided by the total expected number of units bought by high valuation consumers) is increasing in λ. This means that more high valuation consumers participate in auctions as λ increases. As the delay cost w increases (moving horizontally across the table), a higher proportion of auction sales go to the low valuation consumers, since participating in the auction becomes too costly, in terms of delay, for the high valuation consumers.

λ: *w*:	**$0.1/day**	**$0.5/day**	**$1/day**	**$2/day**	**$3/day**	**$4/day**
1/day	.09; .05; .39	.09; .06; .38	.08; .06; .39	.07; .07; .4	.07; .07; .4	.06; .08; .4
2/day	.11; .07; .85	.11; .07; .85	.19; .1; .75	.17; .11; .8	.16; .13; .8	.23; .2; .6
5/day	.29; .17; 2.1	.27; .16; 2.1	.66; .2; 1.7	.71; .29; 1.6	.74; .4; 1.5	.67; .48; 1.5
10/day	.56; .33; 4.2	.56; .33; 4.2	1.52; .34; 3.3	1.7; .58; 2.9	1.71; .86; 2.7	1.61; 1.1; 2.6
20/day	1.16; .68; 8.4	1.08; .64; 8.5	3.63; .65; 6.0	3.8; 1.2; 5.4	3.66; 1.77; 5.1	3.33; 2.25; 5.1
30/day	1.68; .99; 12.7	1.68; .99; 12.7	5.76; .95; 8.6	5.89; 1.82; 7.9	5.55; 2.73; 7.3	5.13; 3.45; 7.5
40/day	2.15; 1.28;17.0	2.15; 1.28; 17.0	2.15; 1.28; 17.0	8.15; 2.56; 9.8	7.47; 3.67; 9.7	6.84; 4.59; 10.0
50/day	3.; 1.79; 21.0	3.; 1.79; 21.0	3.; 1.79; 21.0	10.23; 3.2; 12.3	9.39; 4.61; 12.1	8.55; 5.73; 12.4
60/day	3.75; 2.24;25.0	3.75; 2.24; 25.0	3.75; 2.24; 25.0	12.3; 3.84; 14.7	11.31; 5.55; 14.5	10.35; 6.93; 14.8

Table 3.5: Allocation of items to consumers per 24-hour period (*Expected number of units won by high valuation*; *Expected number of units won by low valuation*; *Expected number bought at posted price*), for various values of consumer arrival rate, λ, and delay cost, *w*.

Figures 3.4 and 3.5 further show the changes in the optimal design, based on the numerical results in Table 3.3. From Figure 3.4 we see that when w is such that it is optimal to set the auction length to the maximum ($T = H$), the optimal lot size increases with the arrival rate at a relatively constant rate, enabling the seller to capture more consumers (to sell more units via auction per unit time). When the arrival rate is low, the size of the auction lot decreases to one unit. Figure 3.5 reveals that for small values of w, when it is optimal to have one-unit auctions, the optimal auction length is decreasing with the arrival rate. As the arrival rate increases, the shorter auctions enable the seller to capture more consumers per unit time.

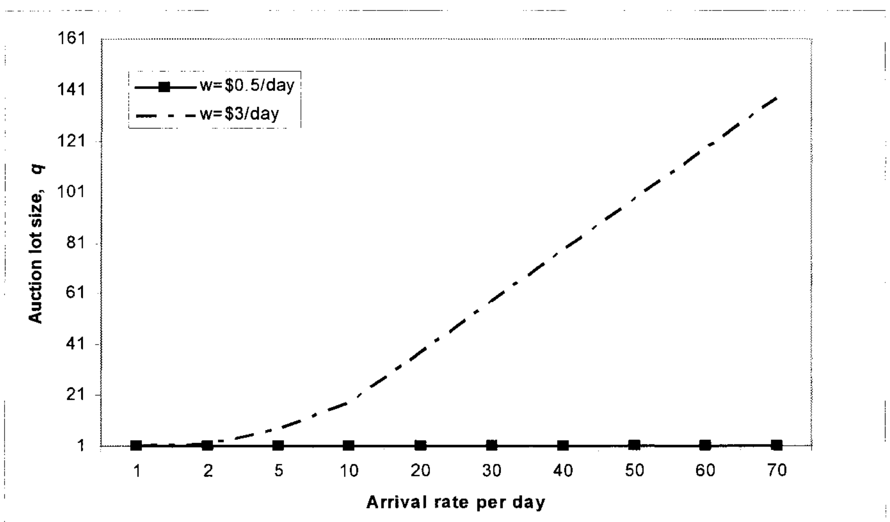

Figure 3.4: The optimal auction lot size as a function of the arrival rate and delay cost, w.

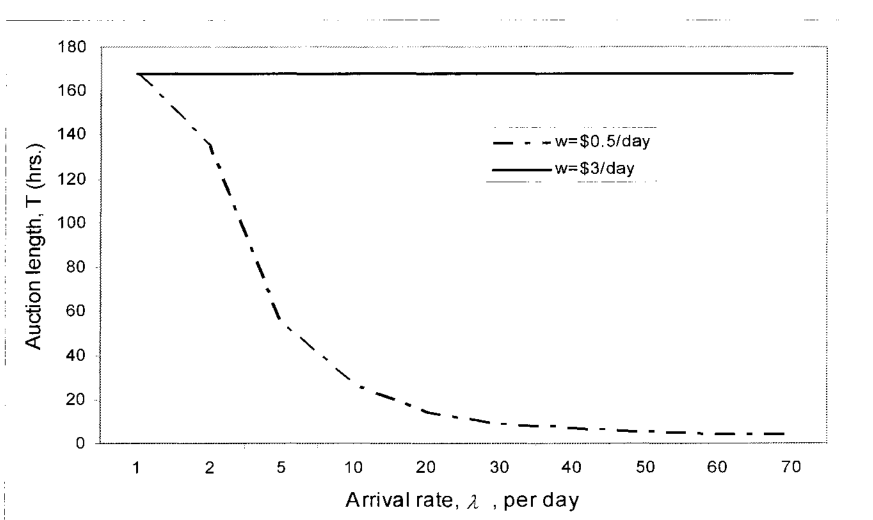

Figure 3.5: The optimal auction length as a function of the arrival rate and delay cost, w.

3.3.2 Revenue Comparison

We now compare the revenue generated by four different selling approaches. The first approach is the optimal management of the dual channel as modeled in this research. In the second approach, the auction and posted price channels are managed independently. In this naïve approach, the posted price is set as if there were no auction channel, and the auction channel design parameters T and q are selected as if there were no posted price channel (see Appendix for detailed results). The third approach is to use only an auction to sell goods, with T and q determined as in the independent approach. The fourth approach is to sell using only a posted price channel.

In Figure 3.6 we plot the percentage revenue increase relative to the revenue from using only posted price for these approaches, as functions of the arrival rate λ, for w= \$1/day. In Figure 3.7, we plot the same as functions of the delay cost for $\lambda = 10$/day.

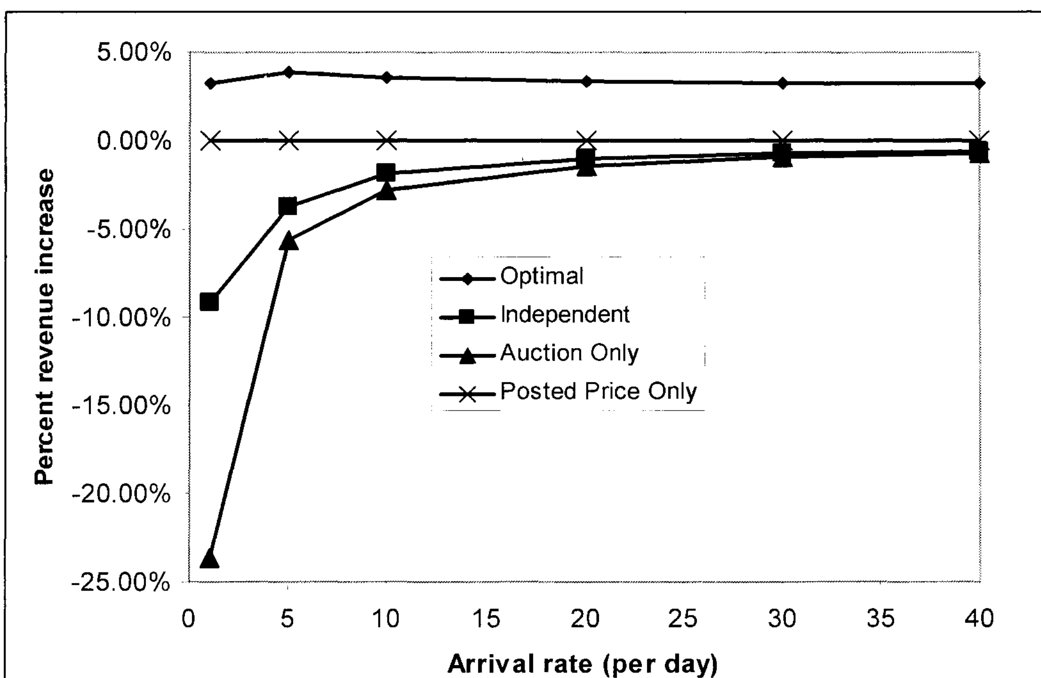

Figure 3.6: Revenue increase relative to revenue from using only posted price as a function of arrival rate, with delay cost w = \$1/day.

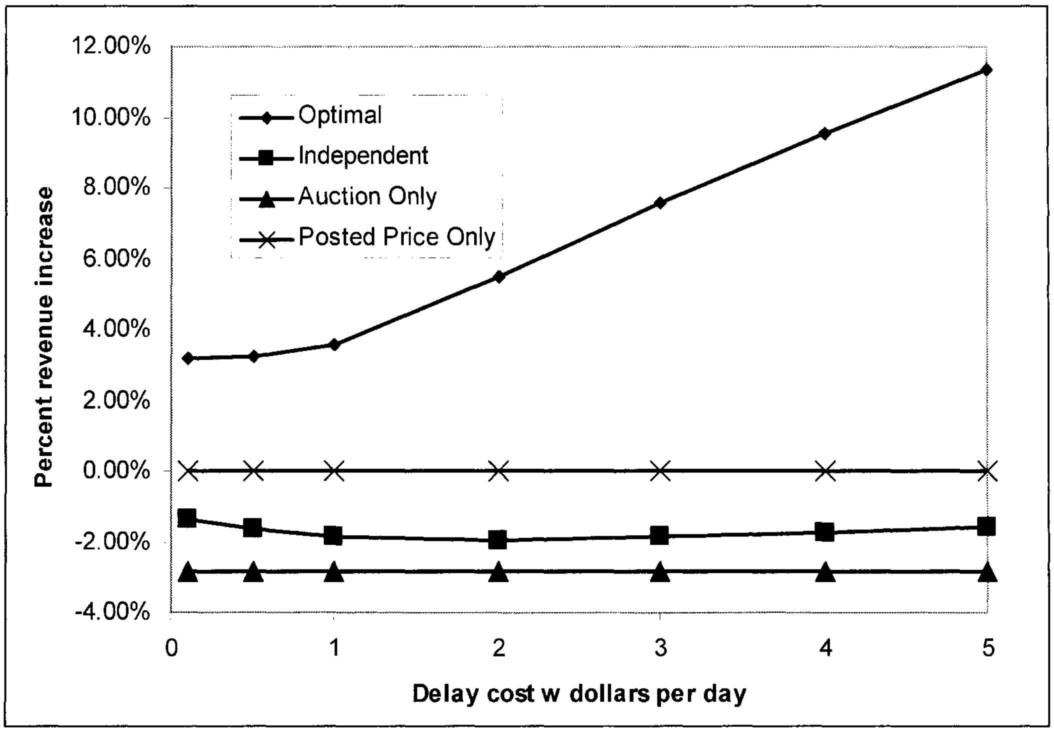

Figure 3.7: Revenue increase relative to posted price as a function of the delay cost, w, with arrival rate $\lambda = 10$/day.

The dual channel's revenue is consistently greater than the posted price revenue, which indicates that when managed optimally the dual channel can effectively serve the low valuation consumers without cannibalizing the high valuation consumers too much. On the other hand, when the channels are managed independently, there is too much cannibalization, and revenues are even worse than when there is just a posted price channel.

In Figure 3.6, we see that when arrival rates are low it is most important for the seller to optimize the two selling channels jointly. In Figure 3.7, we see that the seller's incentive to add auctions is greater when the delay cost perceived by consumers, w, is high. In such circumstances, more effective segmentation of consumers can be achieved, because the seller can sell more via auction without losing too much posted price revenue, and it becomes optimal to increase the auction length and lot size. Many firms have experimented with selling their products online, and many have experimented with online auctions. Our results suggest that unless they jointly manage their online offerings, these firms may find that auctions reduce their revenues. In Figure 3.7 we see potential revenue loss of 5% to 13% when comparing the independent channels design with the optimal dual channel design.

3.4 Concluding Remarks

It is possible to observe many firms selling the same or very similar goods online using posted prices and auctions simultaneously. Our research explains this phenomenon by showing how posted price, auction lot size, and auction duration can be used to segment consumers between an online auction channel and posted price channel to increase a seller's revenue. This allows the auction to be used to capture customers who were priced out by the posted price while mitigating the effects of cannibalization of the posted price channel. Numerical experiments show that the dual channel can significantly outperform a lone posted price channel; we see revenue increases ranging from 2.4% to greater than 10% which are considered significant improvements in retail sales. We show even greater benefits (4% to greater than 13%) over a naïve approach to managing the two channels that optimizes each independently.

Balancing the need to avoid cannibalizing the posted price channel with the opportunity to exploit the potential of the auction channel may seem daunting to a manager. Interestingly, our analysis shows that managing the dual channel optimally may be simpler than it seems. We find that as long as arrival rates are reasonably high, there are only two dominant strategies for managing the dual channel, and in both the posted price is set higher than when there is no parallel auction. One strategy is to offer successive one-unit auctions parallel to the posted price. This strategy appears to be optimal for a wide range of parameter values and is indeed commonly observed in practice. The second strategy is to offer long auctions and it becomes optimal as consumers' delay cost increases. The choice of the optimal strategy depends upon the customer traffic to the website, λ, and the delay cost associated with the product being sold, w.

Our model contains several unique features. We model consumers as making their auction participation decision using an estimate of their expected discount. Their participation decision is directly influenced by the quantity being auctioned and expectations about the number of other bidders. That is, bidders know that if they face greater competition in the auction, the discount over the posted price channel will be smaller. The result is a very realistic and rich portrayal of bidder behavior.

3.5 Proofs

Proof of Lemma 3.1

The auction mechanism awards the items to the bidders with the q highest bids, and each pays a price equal to the highest losing bid. For each player i, define his type as his valuation V_i, and his action as his bid b_i. A strategy is a function from the type space $[\underline{v},\bar{v}]$ to the action space, $B = [0,\infty)$[4]. See Gibbons (1992, Chapter 3) for a detailed definition of static Bayesian games.

Define $b_{-i} = (b_1, b_2, \ldots b_{i-1}, b_{i+1}, \ldots b_N)$ as the vector of actions by the N-1 players besides player i. The payoff received by player i for each combination of actions that could be chosen by the other players, b_{-i}, is given by:[5]

$$U_i(b_i, b_{-i}; V_i) = \begin{cases} \theta[V_i - p_a(b_i, b_{-i})] + (1-\theta)[V_i - p] & \text{if } V_i \geq p \\ \theta[V_i - p_a(b_i, b_{-i})] & \text{if } V_i < p \end{cases}, \qquad (3.9)$$

where $p_a(b_i, b_{-i})$ is the resulting auction price, given by the q+1 highest bid when there are more than q bids, and by the seller's reserve price otherwise, and θ is the probability player i gets an item in the auction. By definition, $\theta \in [0,1]$, and clearly $\theta = 1$ if $b_i > p_a(b_i, b_{-i})$, and $\theta = 0$ if $b_i < p_a(b_i, b_{-i})$ or if $b_i = p_a(b_i, b_{-i})$ and b_i is the only bid that equals the auction's closing price. When $b_i = p_a(b_i, b_{-i})$ and b_i is not the only bid that equals the closing price, the specific expression for θ depends on the tie-breaking rule in use, but regardless of the tie-breaking rule it is still true that $\theta \in [0,1]$, and this is the only property of θ used in the proof.

Next we show that when player i values the item for more than its posted price, $V_i \geq p$, he can do no better than bidding p. That is, for any other action $b'_i \neq p$, for any number

[4] When the current lowest bid required to win is listed, the action space can be bounded from below by that price (choosing not to bid dominates a bid lower than the current lowest bid required to win).

[5] We do not include any cost of delay because once the consumer decides to bid, this cost is sunk. Here, we assume that the decision to bid has already been made, and the search is for a bidding strategy.

of bidders N and combination of other bidders' actions b_{-i}, $U_i(p, b_{-i}; V_i) \geq U_i(b'_i, b_{-i}; V_i)$, with strict inequality for some b_{-i}.

We divide the space of feasible b_{-i} into three exclusive sets (columns in Table 3.6): the first set includes instances of b_{-i} that together with $b_i = p$ result in an auction price that is higher than p; the second set includes instances of b_{-i} that together with $b_i = p$ result in an auction price that equals p; and the last set includes instances of b_{-i} that together with $b_i = p$ result in an auction price that is lower than p. Table 3.6 shows the payoff matrix for a player of type $V_i \geq p$.

	$b_{-i} : p_a(p, b_{-i}) > p$	$b_{-i}: p_a(p, b_{-i}) = p$	$b_{-i} : p_a(p, b_{-i}) < p$
$b_i = p-\Delta$	$V_i - p$	$V_i - p$	$\theta [V_i - p_a(p, b_{-i})] + [1-\theta][V_i - p]$ with $\theta < 1$ for some b_{-i}
$b_i = p$	$V_i - p$	$V_i - p$	$V_i - p_a(p, b_{-i})$
$b_i = p+\Delta$	$\theta [V_i - p_a(p+\Delta, b_{-i})] + [1-\theta][V_i - p]$ with $\theta > 0$ for some b_{-i}	$\theta [V_i - p_a(p+\Delta, b_{-i})] + [1-\theta][V_i - p]$ with $\theta > 0$ and $p_a(p+\Delta, b_{-i}) > p$ for some b_{-i}	$V_i - p_a(p, b_{-i})$

Table 3.6: Payoff matrix for player of type $V \geq p$

For any $\Delta > 0$ and any b_{-i}, $U_i(p, b_{-i}; V_i) \geq U_i(p+\Delta, b_{-i}; V_i)$ and $U_i(p, b_{-i}; V_i) \geq U_i(p-\Delta, b_{-i}; V_i)$ and there is some b_{-i} for which these inequalities are strict. A detailed analysis follows.

Payoffs when $b_i = p$

- When b_{-i} is such that $p_a(p, b_{-i}) > p$, player i does not win the item in the auction and purchases it for the posted price, with payoff $V_i - p$.
- When b_{-i} is such that $p_a(p, b_{-i}) = p$, player i wins the item in the auction with probability $0 \leq \theta \leq 1$. If he wins the item, he pays p, and if he does not win, he purchases it for p. Either way his payoff is $V_i - p$.
- When b_{-i} is such that $p_a(p, b_{-i}) < p$, player i wins the item in the auction with payoff $V_i - p_a(p, b_{-i}) > V_i - p$.

Payoffs when $b'_i = p+\Delta$

- When b_{-i} is such that $p_a(p, b_{-i}) > p$, player i's payoff as a function of b_{-i} is given by $\theta [V_i - p_a(p+\Delta, b_{-i})] + [1-\theta][V_i - p]$, where $p_a(p+\Delta, b_{-i}) \geq p_a(p, b_{-i}) > p$ and $\theta \in [0,1]$. The payoff thus is never higher than the payoff when he bids p. When b_{-i} is such that

$p_a(p, b_{-i}) < p+\Delta$, player i wins the item in the auction and pays $p_a(p+\Delta, b_{-i}) \geq p_a(p, b_{-i}) > p$, so his payoff is less than his payoff from bidding p.

- When b_{-i} is such that $\boldsymbol{p_a(p, b_{-i}) = p}$, player i's payoff as a function of b_{-i} is given by $\theta\,[V_i - p_a(p+\Delta, b_{-i})] + [1-\theta][V_i - p]$, where $p_a(p+\Delta, b_{-i}) \geq p_a(p, b_{-i}) = p$ and $\theta \in [0,1]$. The payoff is thus never higher than the payoff when he bids p. Furthermore, for b_{-i} such that $p+\Delta > p_a(p+\Delta, b_{-i}) > p$, player i wins the item in the auction and pays $p_a(p+\Delta, b_{-i}) > p$, so his payoff is less than his payoff from bidding p.
- When b_{-i} is such that $\boldsymbol{p_a(p, b_{-i}) < p}$, player i wins the item in the auction with payoff $V_i - p_a(p, b_{-i})$, the same payoff as when he bids p, since $p_a(p+\Delta, b_{-i}) = p_a(p, b_{-i})$.

Notice that $U_i(p, b_{-i}; V_i) \geq U_i(p+\Delta, b_{-i}; V_i)$ for every feasible vector b_{-i}, with strict inequality for some instances of b_{-i}. We conclude that $b'_i = p+\Delta$ is weakly dominated by $b_i = p \;\forall\; \Delta > 0$.

Payoffs when $b'_i = p - \Delta$

- When b_{-i} is such that $\boldsymbol{p_a(p, b_{-i}) > p}$, player i does not win the auction and thus purchases the item for the posted price with payoff of $V_i - p$, which is the same as when he bids p.
- When b_{-i} is such that $\boldsymbol{p_a(p, b_{-i}) = p}$, player i does not win the auction and thus purchases the item for the posted price with payoff of $V_i - p$, which is the same as when he bids p.
- When b_{-i} is such that $\boldsymbol{p_a(p, b_{-i}) < p}$, player i's payoff as a function of b_{-i} is given by $\theta[V_i - p_a(p-\Delta, b_{-i})] + [1-\theta][V_i - p]$, where $\theta > 0$ only if $p_a(p-\Delta, b_{-i}) = p_a(p, b_{-i})$ (if player i lowers the auction price by bidding less than p, he will be the highest losing bid). The payoff thus is never higher than the payoff from bidding p and is strictly lower for b_{-i} such that $\theta < 1$.

We note that $U_i(p, b_{-i}; V_i) \geq U_i(p-\Delta, b_{-i}; V_i)$ with strict inequality for some instances of b_{-i}. We conclude that $b'_i = p-\Delta$ is weakly dominated by $b_i = p$. Since the action $b_i = p$ weakly dominates every other action in B, it is a weakly dominant action for a bidder with $V_i > p$ [Mass-Colell et al. 1995, Chapter 8 pp. 238].

In a similar way we can prove that $b_i = V_i$ is a weakly dominant action when $V_i < p$. Table 3.7 shows that for any number of bidders N and for any combination of bidders' actions b_{-i}, player i cannot do better than bidding V_i by bidding $V_i - \Delta$ or $V_i + \Delta$, for any $\Delta > 0$, and for some instances of b_{-i} he does strictly worse.

	$b_{-i} : p_a(V_i, b_{-i}) > V_i$	$b_{-i} : p_a(V_i, b_{-i}) = V_i$	$b_{-i} : p_a(V_i, b_{-i}) < V_i$
$b_i = V_i - \Delta$	0	0	$\theta[V_i - p_a(V_i, b_{-i})]$ with $\theta < 1$ for some b_{-i}
$b_i = V_i$	0	0	$V_i - p_a(V_i, b_{-i})$
$b_i = V_i + \Delta$	$\theta\,[V_i - p_a(v_i+\Delta, b_{-i})] \leq 0$ with $\theta > 0$ for some b_{-i}	$\theta[V_i - p_a(V_i+\Delta, b_{-i})] \leq 0$	$V_i - p_a(V_i, b_{-i})$

Table 3.7: Payoff matrix for a player of type $V < p$

Since the strategy $b(V) = \begin{cases} V & \text{for } V < p \\ p & \text{for } V \geq p \end{cases}$ satisfies $U_i\,(b(V_i), b_{-i}\,; V_i) \geq U_i\,(b'(V_i), b_{-i}\,; V_i)$ for every feasible vector b_{-i}, with strict inequality for some instances of b_{-i}, it is a weakly dominant strategy. ■

Proof of Proposition 3.1

Decision rule (3.5), "participate in the auction if and only if $\Pr(win|p)p - E[auction\_payment \mid p] \geq wt^e$", is derived from rearranging the condition $U_+^A(V) \geq U_+^B(V)$ and assuming that bidders bid according to the strategy specified in Lemma 3.1. According to decision rule (3.5), a high valuation consumer chooses to participate in the auction if and only if his expected discount from participating in the auction exceeds his delay cost.

We define $Q(x)$ as the probability consumer i wins the auction under some tie-breaking rule when there are x other high valuation bidders. Consumer i's expected discount from participating in the auction (LHS in the above decision rule) is given by

$$p\Pr(win \mid p) - E[auction\_payment \mid p] = p\left(\sum_{x=0}^{q-1}\Pr(N_{/i}^{+} = x) + \sum_{x=q}^{\infty} Q(x)\Pr(N_{/i}^{+} = x)\right) - \left(\begin{array}{l} p\sum_{x=q}^{\infty} Q(x)\Pr(N_{/i}^{+} = x) + R\sum_{x=0}^{q-1}\sum_{y=0}^{q-x-1}\Pr(N_{/i}^{+} = x)\Pr(N^{-} = y) + \\ \sum_{x=0}^{q-1}\sum_{y=q-x}^{\infty}\Pr(N_{/i}^{+} = x)\Pr(N^{-} = y)O\{q-(x+1)+1, y\} \end{array}\right) \tag{3.10}$$

where $N_{/i}^{+}$ is the number of high valuation bidders besides consumer i who have decided to participate in the auction, and N^{-} is the number of low valuation bidders. Notice that the particular tie-breaking rule in use does not affect the results because all the terms that depend on the tie-breaking rule cancel out. Canceling terms, we see that a high valuation consumer finds it optimal to participate in the auction iff

$$p\left(\sum_{x=0}^{q-1}\Pr(N_{/i}^{+}=x)\right)-R\sum_{x=0}^{q-1}\sum_{y=0}^{q-x-1}\Pr(N_{/i}^{+}=x)\Pr(N^{-}=y)-\sum_{x=0}^{q-1}\sum_{y=q-x}^{\infty}\Pr(N_{/i}^{+}=x)\Pr(N^{-}=y)O\{q-x,y\}\geq wt^{e} \tag{3.11}$$

Next we prove that there is a unique symmetric equilibrium in which all high valuation consumers use a threshold policy to choose between buying the item for the posted price and participating in the auction. That is, we show that there is a unique value $\bar{t}$ so that if all other high valuation consumers use the threshold $\bar{t}$ to choose between buying the item for the posted price and bidding in the auction, then any other high valuation customer will find that his best response is to use the same threshold as well.

Notice that the LHS in (3.11) does not depend on the valuation of the consumer making the decision, nor on the remaining time of the auction he observes upon arrival, t^e, since he bids $b=p$ regardless of his V and t^e (Lemma 3.1).

If all other high valuation consumers use the threshold $\bar{t}$, then the number of other high valuation bidders $N_{/i}^{+}$ has a Poisson distribution with rate $\lambda\Pr(V\geq p)\bar{t}$ [6], and the number of low valuation bidders N^{-} has a Poisson distribution with rate $\lambda\Pr(V<p)T$. The LHS of (3.11) is thus given by :

$$D(\bar{t})=\sum_{x=0}^{q-1}\frac{e^{-\lambda\Pr(V\geq p)\bar{t}}\left(\lambda\Pr(V\geq p)\bar{t}\right)^{x}}{x!}\left(p-R\sum_{y=0}^{q-x-1}\Pr(N^{-}=y)-\sum_{y=q-x}^{\infty}\Pr(N^{-}=y)O\{q-x,y\}\right), \tag{3.12}$$

and the consumer under consideration chooses to participate in the auction iff

$$D(\bar{t})\geq wt^{e}, \tag{3.13}$$

6 We assume the consumer making the decision uses arrival rate λ, i.e., he does not use a lower arrival rate to account for his own arrival. This makes sense due to the "memory-less" nature of the exponential distribution (at each point of time the distribution of the time remaining until the next arrival stays the same) and due to PASTA- Poisson Arrivals See Time Average.

where $\Pr(N^- = y) = \frac{e^{-\lambda \Pr(V<p)T}(\lambda \Pr(V < p)T)^y}{y!}$ and is not a function of $\bar{t}$.

We can see that the expected discount from participating in the auction is continuous in the threshold used by all other high valuation bidders; i.e. $D(\bar{t})$ is continuous in $\bar{t}$, since it consists of linear combination of polynomial and exponential functions of $\bar{t}$. Notice that O$\{q\text{-}x, y\}$, the expected value of the q-(x+1)+1 order statistic of y draws from the consumer valuations distribution truncated on $[\underline{v}, p]$, is not a function of $\bar{t}$ and can be treated as a constant. We next show that $D(\bar{t})$ is decreasing in $\bar{t}$.

Denote $f(x,\bar{t}) = \frac{e^{-\lambda \Pr(V \geq p)\bar{t}}(\lambda \Pr(V \geq p)\bar{t})^x}{x!}$ and

$$g(x) = p - R\sum_{y=0}^{q-x-1} \Pr(N^- = y) - \sum_{y=q-x}^{\infty} \Pr(N^- = y)O\{q-x, y\}.$$

Then $D(\bar{t}) = \sum_{x=0}^{q-1} f(x,\bar{t})g(x)$. In the following we suppress the dependence of f on $\bar{t}$.

First we show that g(x), the expected discount from the auction when there are x other high valuation bidders, is decreasing in x. That is, we show that $g(x+1) \leq g(x)$.

$$\begin{aligned} &g(x+1) - g(x) = R\Pr(N^- = q-x-1) - \Pr(N^- = q-x-1)O\{q-x-1, y\} = \\ &\Pr(N^- = q-x-1)(R - O\{q-x-1, y\}) \leq 0 \end{aligned},$$

where the last inequality holds because $O\{q-x-1, y\} \geq R$, since no bids are less than R.

In addition, $g(x) > 0$ for every value of x because $R < p$ and $O\{q\text{-}x, y\} < p$.

Next, using the fact that

$$\frac{\partial f(x,\bar{t})}{\partial \bar{t}} = \begin{cases} \lambda \Pr(V \geq p)\left(f(x-1,\bar{t}) - f(x,\bar{t})\right) & \text{if } x > 0 \\ -\lambda \Pr(V \geq p)e^{-\lambda \Pr(V \geq p)\bar{t}} & \text{if } x = 0 \end{cases},$$

we take the derivative of $D(\bar{t})$ with respect to $\bar{t}$:

$$\frac{\partial D(\bar{t})}{\partial \bar{t}} = \frac{\partial \sum_{x=0}^{q-1} f(x)g(x)}{\partial \bar{t}} = \frac{\partial f(0)g(0)}{\partial \bar{t}} + \lambda \Pr(V \geq p) \sum_{x=1}^{q-1} g(x)\left(f(x-1) - f(x)\right) =$$

$$\lambda \Pr(V \geq p)\left(-g(0)e^{-\lambda \Pr(V \geq p)\bar{t}} + \sum_{x=1}^{q-2} f(x)\left(g(x+1) - g(x)\right) + g(1)f(0) - g(q-1)f(q-1) \right) < 0 .$$

The last inequality holds because $g(0) > 0$, $g(q\text{-}1)f(q\text{-}1) > 0$, $g(x{+}1) < g(x)$ as shown above, and $g(1)f(0) = g(1)e^{-\lambda \Pr(V \geq p)\bar{t}} < g(0)e^{-\lambda \Pr(V \geq p)\bar{t}}$.

Notice that $t^e \leq T$, and so we can consider only values of $\bar{t}$ that do not exceed T. If $D(T) \geq wT$, then the unique symmetric threshold equilibrium is given by the threshold T. To prove this point, assume on the contrary that there is a symmetric threshold equilibrium s, such that $s < T$. Then, if all other high valuation consumers use the threshold s, a high valuation consumer's best response is to use the threshold T. That is, because $D(\bar{t})$ is decreasing in $\bar{t}$, $D(s) > D(T) \geq wT$ and the consumer's expected discount from bidding when the remaining time of the auction is T, is larger than his delay cost. Hence, $s < T$ can not be a symmetric threshold equilibrium when $D(T) \geq wT$. If all other high valuation consumers use the threshold T, then because $D(T) \geq wT$, a high valuation consumer's best response is to use the threshold T.

If $D(T) < wT$, then T can not be a symmetric threshold equilibrium, because if all other high valuation consumers use the threshold T, a high valuation consumer's best response is to use a threshold smaller than T. However, we now show that if $D(T) < wT$, then there is a unique symmetric threshold equilibrium, $0 < \bar{t} < T$.

Assume $D(T) < wT$ and that all other high valuation consumers use the threshold $0 < \bar{t} < T$. A high valuation consumer chooses to participate in the auction iff $D(\bar{t}) \geq wt^e$, and since $D(\bar{t})$ is not a function of t^e, the consumer uses the threshold t^*, given by the solution of $D(\bar{t}) = wt^*$. $D(\bar{t}) > 0$, since there is a positive probability that the expected auction price is strictly less than p, and so t^* has to be positive. In addition, since $D(\bar{t})$ is continuous and strictly decreasing in $\bar{t}$, there exists a unique value of $\bar{t}$ such that $D(\bar{t}) = w\bar{t}$, and this value has to be smaller than T, since $D(T) < wT$. That value of $\bar{t}$ is the unique symmetric threshold equilibrium. ■

Proof of Proposition 3.2

To simplify notation, we define $\lambda^+ = \lambda \Pr(V \geq p)t$ and $\lambda^- = \lambda \Pr(V < p)T$. Note that λ^+ is continuous and strictly increasing in *t* while λ^- is independent of *t*. $D_h(t)$, the expected auction discount over the posted price for a high valuation bidder who uses Heuristic 1 and assumes all other high valuation consumers use the threshold *t*, is given by

$$\begin{aligned}&\rho(t)\gamma d(\lceil \lambda \Pr(V \geq p)t+1\rceil, \lceil \lambda \Pr(V < p)T\rceil)+\\&(1-\rho(t))(1-\gamma)d(\lfloor \lambda \Pr(V \geq p)t+1\rfloor, \lfloor \lambda \Pr(V < p)T\rfloor)+\\&(1-\rho(t))\gamma d(\lfloor \lambda \Pr(V \geq p)t+1\rfloor, \lceil \lambda \Pr(V < p)T\rceil)+\\&\rho(t)(1-\gamma)d(\lceil \lambda \Pr(V \geq p)t+1\rceil, \lfloor \lambda \Pr(V < p)T\rfloor)\end{aligned} \tag{3.14}$$

where *d*(*x*, *y*) is the expected discount over the posted price for a high valuation consumer who participates in an auction with a total of *x* high valuation bidders (including himself) and *y* low valuation bidders, as given in Equation 3.15:

$$d(x,y) = \begin{cases} p - O\{q-x+1, y\} & \text{if } x \leq q \text{ and } x+y > q \\ 0 & \text{if } x > q \\ p - R & \text{if } x+y \leq q \end{cases}. \tag{3.15}$$

Since we use the uniform distribution for each consumer's valuation of the item, in our numerical experiments, $O(q-x+1, y) = \underline{v} + (p-\underline{v})\dfrac{y-(q-x)}{y+1}$ [see Pinker et al. 2000].

We next show that there is a unique (symmetric) equilibrium in which all high valuation consumers use the threshold $\bar{t}_h$. That is, when $D_h(T) < wT$, there is a unique value $t < T$ satisfying $D_h(t) = wt$, and we denote it by $\bar{t}_h$, and when $D_h(T) \geq wT$, $\bar{t}_h = T$.

As in proof of Proposition 3.1, since *wt* is strictly increasing in *t*, it is sufficient to show that $D_h(t)$ is continuous and non-increasing in *t* and that $D_h(t) > 0$. We examine $D_h(t)$ over five ranges of *t* values, starting from the right.

Range 1: $t \geq \dfrac{q}{\lambda \Pr(V > p)}$. In this range we have $\lfloor \lambda^+ + 1 \rfloor > q$ and so $D_h(t)$ equals 0.

Range 2: $\dfrac{q-1}{\lambda \Pr(V \geq p)} < t < \dfrac{q}{\lambda \Pr(V \geq p)}$. In this range, $\lceil \lambda^+ + 1 \rceil = q+1 > q$, but $\lfloor \lambda^+ + 1 \rfloor = q$, so $D_h(t)$ is given by

$$\gamma(1-\rho(t))d\left(\lfloor\lambda^{+}+1\rfloor,\lceil\lambda^{-}\rceil\right)+(1-\gamma)(1-\rho(t))d\left(\lfloor\lambda^{+}+1\rfloor,\lfloor\lambda^{-}\rfloor\right)=$$

$\gamma\left(1-\left(\lambda^{+}-\lfloor\lambda^{+}\rfloor\right)\right)\frac{p}{\lceil\lambda^{-}\rceil+1}+(1-\gamma)\left(1-\left(\lambda^{+}-\lfloor\lambda^{+}\rfloor\right)\right)\frac{p}{\lfloor\lambda^{-}\rfloor+1}$, which is continuous and linearly decreasing in t (since λ^{+} is linearly increasing in t and $\lfloor\lambda^{+}\rfloor=q-1$ for this entire range of t values). We next show that when t approaches the upper bound of this range the expected auction discount approaches the value of the expected auction discount at the lower bound of Range 1, and that when t approaches the lower bound of this range the expected auction discount approaches its value at the upper bound of Range 3.

$$\mathop{Lim}_{t\to\frac{q}{\lambda\Pr(V\geq p)}\text{ from below}} \mathrm{D_h}(t)=$$

$$\mathop{Lim}_{\lambda^{+}\to q}\gamma\left(1-\left(\lambda^{+}-(q-1)\right)\right)\left(\frac{p}{\lceil\lambda^{-}\rceil+1}\right)+(1-\gamma)\left(1-\left(\lambda^{+}-(q-1)\right)\right)\left(\frac{p}{\lfloor\lambda^{-}\rfloor+1}\right)=0$$

Using $\mathop{Lim}_{\lambda^{+}\to\lceil\lambda^{+}\rceil}\left(\lambda^{+}-\lfloor\lambda^{+}\rfloor\right)=1$.

$$\mathop{Lim}_{t\to\frac{q-1}{\lambda\Pr(V\geq p)}\text{ from above}} \mathrm{D}_h(t)=$$

$$\mathop{Lim}_{\lambda^{+}\to q-1}\gamma\left(1-\left(\lambda^{+}-(q-1)\right)\right)\left(\frac{p}{\lceil\lambda^{-}\rceil+1}\right)+(1-\gamma)\left(1-\left(\lambda^{+}-(q-1)\right)\right)\left(\frac{p}{\lfloor\lambda^{-}\rfloor+1}\right)=$$

$$\frac{\gamma p}{\lceil\lambda^{-}\rceil+1}+\frac{(1-\gamma)p}{\lfloor\lambda^{-}\rfloor+1}=D_h\left(\frac{q-1}{\lambda\Pr(V\geq p)}\right)$$

Range 3: $Max\left(0,\frac{q-\lfloor\lambda^{-}\rfloor}{\lambda\Pr(V\geq p)}\right)\leq t\leq\frac{q-1}{\lambda\Pr(V\geq p)}$.

In this range, $\lceil\lambda^{+}+1\rceil\leq q$ and $\lfloor\lambda^{+}+1\rfloor+\lfloor\lambda^{-}\rfloor>q$. Therefore, $D_h(t)$ is given by

$$\gamma\left(\lambda^{+}-\lfloor\lambda^{+}\rfloor\right)\left(\frac{p(q-\lceil\lambda^{+}\rceil)}{\lceil\lambda^{-}\rceil+1}\right)+\gamma\left(1-\left(\lambda^{+}-\lfloor\lambda^{+}\rfloor\right)\right)\left(\frac{p(q-\lfloor\lambda^{+}\rfloor)}{\lceil\lambda^{-}\rceil+1}\right)+$$

$$(1-\gamma)\left(\lambda^{+}-\lfloor\lambda^{+}\rfloor\right)\left(\frac{p(q-\lceil\lambda^{+}\rceil)}{\lfloor\lambda^{-}\rfloor+1}\right)+(1-\gamma)\left(1-\left(\lambda^{+}-\lfloor\lambda^{+}\rfloor\right)\right)\left(\frac{p(q-\lfloor\lambda^{+}\rfloor)}{\lfloor\lambda^{-}\rfloor+1}\right)=$$

$$=\left(\frac{\gamma p}{\lceil\lambda^{-}\rceil+1}+\frac{(1-\gamma)p}{\lfloor\lambda^{-}\rfloor+1}\right)(q-\lambda^{+}).$$

In this range of t values, $D_h(t)$ is continuous and linearly decreasing in t because λ^{+} is linearly increasing and continuous in t.

Range 4: $Max\left(0, \frac{q-\lceil \lambda^- \rceil - 1}{\lambda \Pr(V \geq p)}\right) < t < Max\left(0, \frac{q-\lfloor \lambda^- \rfloor}{\lambda \Pr(V \geq p)}\right)$. It is easy to show that in this range of t values $D_h(t)$ is continuous and linearly decreasing in t whenever λ^+ is non-integer. Hence, here we focus on showing that $D_h(t)$ is continuous when λ^+ is an integer for t in the above range, and at the upper and lower bounds of this range. When λ^+ approaches an integer from below, $Lim(\lambda^+ - \lfloor \lambda^+ \rfloor) = 1$, so $Lim\ \rho(t) = 1$. Therefore,

$$Lim_{t \xrightarrow{below} \frac{q-\lfloor \lambda^- \rfloor}{\lambda \Pr(V \geq p)}} D_h(t) = \gamma\left(p \frac{\lfloor \lambda^- \rfloor}{\lceil \lambda^- \rceil + 1}\right) + (1-\gamma)\left(p \frac{\lfloor \lambda^- \rfloor}{\lfloor \lambda^- \rfloor + 1}\right) =$$

$$\left(\frac{\gamma p}{\lceil \lambda^- \rceil + 1} + \frac{(1-\gamma)p}{\lfloor \lambda^- \rfloor + 1}\right)\lfloor \lambda^- \rfloor = D_h\left(\frac{q-\lfloor \lambda^- \rfloor}{\lambda \Pr(V \geq p)}\right).$$

When λ^+ approaches an integer from above, $Lim(\lambda^+ - \lfloor \lambda^+ \rfloor) = 0$, so $Lim\ \rho(t) = 0$.

Therefore,

$$Lim_{t \xrightarrow{above} \frac{q-\lceil \lambda^- \rceil - 1}{\lambda \Pr(V \geq p)}} \mathrm{D_h(t)} =$$

$$Lim_{t \xrightarrow{above} \frac{q-\lceil \lambda^- \rceil - 1}{\lambda \Pr(V \geq p)}} \rho(t)\gamma d(\lceil \lambda^+ + 1 \rceil, \lceil \lambda^- \rceil) + (1-\rho(t)\gamma)(p-R) = P - R\ .$$

If λ^- is an integer, then there are no t inside Range 4 such that λ^+ is integer. When λ^- is not an integer, there is only one value of t, in Range 4, for which λ^+ is an integer, namely, $\frac{q-\lceil \lambda^- \rceil}{\lambda \Pr(V \geq p)}$, and

$$D_h\left(\frac{q-\lceil \lambda^- \rceil}{\lambda \Pr(V \geq p)}\right) = \gamma d(q - \lceil \lambda^- \rceil + 1, \lceil \lambda^- \rceil) + (1-\gamma)(p-R)\ .$$

We show now that $D_h(t)$ is continuous at this value of t:

$$Lim_{t \xrightarrow{below} \frac{q-\lceil \lambda^- \rceil}{\lambda \Pr(V \geq p)}} D_h(t) = Lim_{\lambda^+ \xrightarrow{below} q-\lceil \lambda^- \rceil} \gamma d(\lceil \lambda^+ + 1 \rceil, \lceil \lambda^- \rceil) + (1-\gamma)d(\lceil \lambda^+ + 1 \rceil, \lfloor \lambda^- \rfloor) =$$

$$\gamma d(q - \lceil \lambda^- \rceil + 1, \lceil \lambda^- \rceil) + (1-\gamma)(p-R) = D_h\left(\frac{q-\lceil \lambda^- \rceil}{\lambda Pr(V \geq p)}\right)$$

$$\lim_{t \xrightarrow{above} \frac{q-\lceil \lambda^- \rceil}{\lambda \Pr(V \geq p)}} D_h(t) = \lim_{\lambda^+ \xrightarrow{above} q-\lceil \lambda^- \rceil} (1-\gamma) d\left(\lfloor \lambda^+ +1 \rfloor, \lfloor \lambda^- \rfloor\right) + \gamma d\left(\lfloor \lambda^+ +1 \rfloor, \lceil \lambda^- \rceil\right)$$

$$= \gamma d\left(q - \lceil \lambda^- \rceil + 1, \lceil \lambda^- \rceil\right) + (1-\gamma)(p-R) = D_h\left(\frac{q - \lceil \lambda^- \rceil}{\lambda Pr(V \geq p)}\right).$$

Range 5: $0 \leq t \leq Max\left(0, \frac{q - \lceil \lambda^- \rceil - 1}{\lambda^-}\right)$. In this range, $D_h(t) = (p\text{-}R)$, since for such values of t the total number of bidders does not exceed q in each of the four terms in $D_h(t)$ and $d(x, y) = p\text{-}R$ when $(x + y) \leq q$.

Since $D_h(t)$ is continuous and constant or linearly decreasing in t in each of the above five ranges of t values, and continuous at the edges of each range, we conclude that $D_h(t)$ is continuous and non-increasing in t for every $t \geq 0$. In addition, $D_h(0) = (p-R) > 0$. Hence, $D_h(t) = wt$ is a fixed-point equation with a unique solution. The rest of the proof is similar to the proof of Proposition 3.1■

Chapter 4

Suppliers Use of Spot Markets in Industries with Forward Contracts

4.1 Introduction

Electronic B2B spot markets have captured the attention of many academic researchers and business pundits. Many predicted that eventually B2B spot markets would eliminate forward contracting, at least for the procurement of commodities, because spot markets allow buyers to delay the procurement decision until they have more information about demand, and enable them to exploit competition between suppliers to reduce the procurement cost. Yet the failure of many B2B spot markets was clear evidence that contracting and long term relationships with suppliers have merit, which, for many buyers, outweighs the benefits of a potentially lower spot price. Despite high expectations, many spot markets have only a small number of suppliers and limited liquidity, as *The Economist* notes:

> *In a few other industries, such as steel, B2B exchanges are also starting to make some headway in trading contracts, having realized that spot markets, though easy to enter, are usually too small.... Too many B2B exchanges focused on the spot markets in their industries and are now paying the price. By focusing on the exception, rather than the rule, they were bound to remain fringe players, starved of liquidity and ignored by most of the big firms in their industry, which continues negotiating contracts with each other as before.*
> ["The Container Case", *The Economist*, Oct 2000]

Researchers argue that buyers optimize procurement over both channels as a risk management technique, since there is a tradeoff between contracting earlier, when demand is uncertain, for a known unit price, and waiting to buy on the spot market, after demand is realized, for an uncertain unit price. Consequently, the more buyers contract in advance, the less they are likely to later buy on the spot market, and demands on the two channels (contracts and spot market) are negatively correlated. Nevertheless, even if demands on the two channels originate from different buyers, such demands are not necessarily independent. For example, we would expect to have a positive correlation between the spot market demand and the contracted demand, when the two independent groups of buyers face the same end-demand stream. In this chapter, we model strategic behavior of suppliers in B2B spot markets and generalize the demand side, allowing for both negative and positive correlation between the spot market demand and the contracted demand.

There are differences among types of suppliers and how they use spot markets. Many use spot markets for inventory liquidation, offering only their excess inventory (what remains after satisfying the contracted demand) on the market. Yet, spot markets are also used by new or small suppliers who do not have loyal customers and forward contracts. Suppliers in this latter group use the spot market as their sole source of business. In this chapter, we model a spot market in a make-to-stock industry with two suppliers. Each supplier belongs to one of two types: type 1 supplier has loyal customers (forward contracts) and, after observing the contracted demand, decides how to split his inventory between his loyal customers and the spot market; type 2 supplier does not have loyal customers and produces only for the spot market. The demand from loyal customers and the spot price are unknown at the time of production. The spot market clears with the quantities offered by the suppliers, who compete as a Cournot duopoly.

Examining a market with one supplier of each type, we find five feasible production quantity equilibria in which the supplier that has forward contracts sells only the inventory remaining after satisfying demand from his contracts, on the spot market. The distributions of the spot market price and contracted demand, the contracted unit price, the production cost and the correlation between the demands determine which of the five equilibria can prevail. When the expected spot market price is high and production cost is low, the type 1 supplier participates in the spot market whether demand from his forward contracts is high or low. That is, he uses the spot market as an additional selling channel. We show that in such equilibrium, the contracted demand of the type 1 supplier has no effect on the production quantity or profit of the supplier that works solely on the spot market. In addition, the expected spot market supply is the same as in a market with two type 2 suppliers. When the expected spot market price and production cost are such that in equilibrium the type 1 supplier participates in the spot market only when demand from his contracts is low, the forward contracts affect the production quantity and profit of the supplier that works solely on the spot market. In this chapter we focus on an equilibrium with this last property because we are interested in studying the interaction between different supplier types, and name it the "Liquidation Equilibrium". The Liquidation Equilibrium is the only equilibrium in which the type 1 supplier incurs a penalty cost when contracted demand is high, because his inventory is not sufficient to satisfy a high level of contracted demand. In addition, it is the only equilibrium in which the contracted unit price affects the production quantities. We answer the following research questions: (1) How does the contracted demand of the type-1 supplier affect the production decision and profit of the supplier who sells only on the spot market? (2) Which supplier type benefits more from the existence of the spot market? (3) Which supplier benefits more from an increase in spot market demand? (4) Do spot markets have a negative effect on the service provided to contracting customers?

Our results show that in the Liquidation Equilibrium, when demands on the two channels are independent of each other, the profit and the production quantity of the supplier with no contracts increase as the probability that the type-1 supplier participates in the spot market increases, and the supplier with no contracts is better off if the type-1 supplier eliminates contracting altogether. This result holds when demands are positively correlated, but under certain conditions does not hold with a negative correlation. We find that when the spot market demand is small, the type 1 supplier has a higher incentive to invest in expanding the spot market. When the spot market demand exceeds a threshold size, this situation is reversed and the supplier with no contracts benefits more from making the spot market more prevalent. We show that the supplier with forward contracts benefits from the existence of the spot market more than the supplier with no contracts, and that this result holds with both negative and positive correlation between spot market and contracted demands.

For Liquidation Equilibrium, we prove that suppliers producing only for the spot market gain from working in industries where contracted demand and spot market demand are positively correlated, while suppliers that have forward contracts benefit from working in industries with a negative correlation between demands, as it allows them to better manage risk. In addition, both total industry supply and spot market supply are higher in industries where demands are negatively correlated. Thus, a positive correlation between demands on the two channels is likely to make buyers worse off, while a negative correlation increases industry supply and, generally, improves buyers' welfare. This result agrees with previous research on B2B markets showing that buyers should use both channels.

This chapter is structured as follows. In §4.2, I construct the model for selling to loyal customers and for spot-market clearance, and describe production quantity equilibria. In §4.3, I analyze the Liquidation Equilibrium and examine which supplier type benefits more from the existence of the spot market, and which supplier type is more likely to invest in the spot market in its different stages. In § 4.4, I examine the effect of correlation between spot market demand and contracted demand on the production quantities and spot market supply. I shortly discuss other equilibria in § 4.5 and conclude in § 4.6. Part of the proofs and technical derivations are presented in § 4.7.

4.2 The Model

We examine a make-to-stock industry with two types of suppliers. A supplier of the first type has loyal customers (forward contracts) to whom he sells for a fixed unit price. The demand from these customers is unknown at the time he makes his production decision. After observing the contracted demand, this supplier decides how to split his inventory between his loyal customers and a spot market, where he competes with other suppliers. A supplier of the second type does not have loyal customers (contracts), and thus produces only for the spot market. The spot market price is determined by the total quantity offered by all suppliers. Here we assume that only two suppliers use the spot market and that, unless otherwise specified and without loss of generality, Supplier 1 has loyal customers (is of type 1) and Supplier 2 works only on the spot market (is of type 2).

In the environment we model, production takes place before demand from forward contracts is realized. At the time Supplier 1 makes the production decision, the demand from his loyal customers, D, is believed to be high with probability α and low with probability $(1-\alpha)$. We assume that the contracted unit price, w, is exogenously given and that there is a penalty cost, k, for each unit of unsatisfied demand. Table 4.1 summarizes the model notation.

D	demand from loyal customers, a discrete random variable with support set $\{L, H\}$.
α	probability of high contracted demand.
w	unit price for loyal customers.
k	penalty cost for each unit not delivered to loyal customers.
c	unit production cost.
Q_i	quantity produced by supplier i, $i \in \{1,2\}$.
q_i	quantity offered on the spot market by supplier i, $i \in \{1,2\}$.
p_{MP}	unit price on the spot market.
$\pi_i(Q_i)$	profit of supplier i as function of his production lot.
π_i^L	maximum profit of a type-1 supplier, when there is no spot market.

Table 4.1: Notation for Chapter 4

After observing the contracted demand, Supplier 1 decides how much of his inventory to sell to his loyal customers and how much to offer on the spot market. The contracted demand can serve as a signal regarding the spot market price: high (low) contracted demand can raise (lower) expectations regarding the spot market price, when demands on the two channels are positively (negatively) correlated. Hence, after observing the contracted demand, Supplier 1 updates his belief regarding the distribution of the spot market price.

The spot market clears with the quantities offered by the suppliers, who compete as a Cournot duopoly. The demand curve (describing the spot price as function of quantities demanded) is assumed to be linear with slope normalized to 1. That is, the spot market price is given by

$$p_{MP} = B + d - \sum_{i=1}^{2} q_i \,, \tag{4.1}$$

where p_{MP} is the clearing price and q_i is the quantity offered on the spot market by Supplier i. B and d are in price units, and the coefficient of q_i is 1[price/quantity]. B represents the known component of the demand intercept while d represents the uncertain component (see Tunca, 2002). To allow for dependence between the demands on the two channels, we first define the distribution of d conditional on the value of the contracted demand, D. We model $d|D$ as a random variable from the normal distribution with mean $\beta(D\text{-E}[D])$ and standard deviation σ. Given the above distribution of $d|D$, the random variable d is a mixture of normal distributions (see Greene, W.H., p. 529), with mean $\alpha(H - E[D]) + (1-\alpha)(L - E[D]) = 0$ and variance $\sigma^2 + \beta^2 VAR[D]$. When $\beta = 0$, the spot market demand and the contracted demand are independent of each other, and d is drawn from the standard normal distribution, $N(0, \sigma^2)$. When $\beta > 0$ ($\beta < 0$) the demands on the two channels are positively (negatively) correlated. That is, when $\beta > 0$ we expect the deviation of the contracted demand from its expected value, and the deviation of the spot market demand from its expected value, to be in the same direction: if demand originating from contracts exceeded its expected value, there is high probability that the demand on the spot market would exceed its expected value. For example, if end-user demand for gasoline is rising, firms in the gasoline industry will purchase larger quantities than expected, via both contracts and spot markets. Hence, the demand for gasoline in the two channels is positively correlated. When $\beta < 0$, we expect the deviations from the expected values to be in opposite directions. For example, in an industry with a limited number of buyers, a large buyer might split his demand between the two channels as a risk management technique. In this case observing low contracted demand increases the probability of high spot market demand, and the demands on the two channels are negatively correlated.

Examining the inverse demand curve,

$\sum_{i=1}^{2} q_i = 1\left[\frac{quantity}{price}\right] \times B + 1\left[\frac{quantity}{price}\right] \times d - 1\left[\frac{quantity}{price}\right] \times P_{MP}$, it is clear that B is an indicator for the size of the spot market demand. We denote $B+\beta\,(H\text{-E}[D])$ by B_H and $B+\beta\,(L\text{-E}[D])$ by B_L, and assume that the unit production cost, c, is such that $B > c$, since otherwise a supplier with no contacts does not participate in the spot market.

If α is large, observing high contracted demand should not have a large effect on the expected spot price, since a high level of contracted demand was anticipated. However, observing low contracted demand, which was unlikely, should have a large effect on the expected spot price. Indeed, in our model the adjustment to the expected spot price when observing high contracted demand, $|E[p_{MP} \mid H] - E[p_{MP}]| = |\beta|(1-\alpha)(H - L)$, is decreasing in

α, and the adjustment to the spot price when observing low contracted demand, $|E[p_{MP} \mid L] - E[p_{MP}]| == |\beta|\alpha(H-L)$, is increasing in α. Also, in our model, an increase in the variance of the contracted demand increases the variance of the spot market demand when $\beta \neq 0$. The variance of the contracted demand is increasing in $(H\text{-}L)$, and the difference between the expected spot market price when contracted demand is high and the expected spot market price when contracted demand is low, $|E[p_{MP} \mid H] - E[p_{MP} \mid L]| = |\beta|(H-L)$, increases in $(H\text{-}L)$.

When demands are negatively correlated, the reason is usually that buyers optimize on using both channels. Hence, it is reasonable to assume $\beta \geq -1$, so that the adjustment to the expected spot market demand is not larger than the realized deviation from the expected value of the contracted demand.

The lot offered on the spot market by a supplier with no contracts is limited by the quantity he produced. The expected profit of the supplier with no contracts is given by:

$$\pi_2 (Q_2) = Q_2 (B - Q_2 - \mathrm{E}[q_1]) - cQ_2. \tag{4.2}$$

We assume that suppliers act simultaneously when offering their inventories on the spot market. In addition, a supplier of type 2 cannot observe the realization of the contracted demand of a supplier of type 1, and so when he offers his inventory on the spot market he has the same information as when he made the production decision. Therefore, a supplier with no contracts has no incentive to produce more, or less, than the quantity he plans to offer on the marketplace. Taking the derivative of the profit with respect to Q_2 and solving for the optimal production quantity as a function of $\mathrm{E}[q_1]$, (SOC is satisfied), we find the best response function for a type 2 supplier as given in Equation 4.3.

$$Q_2 (\mathrm{E}[q_1]) = \frac{B - E[q_1] - c}{2} \tag{4.3}$$

The type 1 supplier faces a two stage optimization problem. In the first stage he needs to make the production decision, Q_1, knowing only the distributions of contracted demand and spot market price. In the second stage, after observing the contracted demand, he updates his belief regarding the spot market price, and makes the inventory allocation decision. We start with the second stage. When contracted demand is high, Supplier 1 chooses q_1 that maximizes his expected revenue from both channels, given by $R(q_1|H) = wMin(H, Q_1 - q_1) - k(H - (Q_1 - q_1))^+ + q_1(B_H - Q_2 - q_1)$, subject to $q_1 \leq Q_1$.

Similarly, when contracted demand is low, Supplier 1 chooses q_1 that maximizes his expected revenue as given by

$R(q_1|L) = wMin(L, Q_1 - q_1) - k(L - (Q_1 - q_1))^+ + q_1(B_L - Q_2 - q_1)$, subject to $q_1 \leq Q_1$.

The derivative of Supplier 1's revenue with respect to q_1 is given by:

$$\frac{\partial R(q_1|D)}{\partial q_1} = \begin{cases} B_D - Q_2 - 2q_1 & \text{if } q_1 < Q_1 - D \\ B_D - Q_2 - 2q_1 - (w+k) & \text{else} \end{cases}.$$

In this chapter we limit our attention to equilibria in which the forward contract, specified by w and k, is such that Supplier 1 uses his inventory to satisfy demand from loyal customers (contracts) before offering units on the spot market. That is, we are interested in equilibria in which the optimal amount allocated to the spot market satisfies $q_1 \leq (Q_1\text{-}D)^+$, for both D=H and D=L. Notice that there can not be equilibrium in which Supplier 1 always under-satisfies contracted demand by selling more than $(Q_1\text{-}D)^+$ units on the spot market for any realization of D, since then his profit would be strictly increasing in Q_1, and he would deviate and produce more. However, there are equilibria in which $q_1 > (Q_1\text{-}H)^+$ **or** $q_1 > (Q_1\text{-}L)^+$. Customers that contract in advance can negotiate values of w and k at which Supplier 1 gives them priority over the spot market, for any realization of contracted demand. The result of such negotiation is beyond the scope of this research. We treat w and k as exogenously given, and assume that their sum is high enough to ensure Supplier 1 stays "loyal" to his loyal customers. That is, we assume $\left.\frac{\partial R(q_1 \mid D)}{\partial q}\right|_{q_1=(Q_1-D)^+} \leq 0$, which holds if and only if

$$w+k > \text{Max}(B_H - Q_2 - 2(Q_1 - H)^+,\ B_L - Q_2 - 2(Q_1 - L)). \quad (4.4)$$

If both $Q_1 > H + 0.5(B_H - Q_2)$ and $Q_1 > L + 0.5(B_L - Q_2)$, then Supplier 1 offers less than $(Q_1\text{-}D)^+$ units on the spot market, for every realization of contracted demand. Such values of (Q_1, Q_2) can not be equilibrium, since Supplier 1's profit would be linearly decreasing in Q_1, and he would deviate and produce less. We can therefore assume from now on that the equilibrium production quantities satisfy

$Q_1 \le Max(H + 0.5(B_H - Q_2),\ L + 0.5(B_L - Q_2)) = H + 0.5(B_H - Q_2)$[7], and therefore

$Min(Q_1 - H,\ 0.5(B_H - Q_2)) = Q_1 - H$.

Assuming the production quantities, (Q_1, Q_2), satisfy Condition 4.4, Supplier 1's optimal spot market lot size is given by:

$$q_1(D) = \begin{cases} Min\left(Q_1 - L,\ \dfrac{B_L - Q_2}{2}\right) & \text{if } D = L \\ Max\left(0, Q_1 - H\right) & \text{if } D = H \end{cases}, \qquad (4.5)$$

and his expected revenue from the spot market is given by:

$$R_1(Q_1) = \alpha q_1(H)[B_H - Q_2 - q_1(H)] + (1-\alpha) q_1(L)[B_L - Q_2 - q_1(L)].$$

In the first stage, Supplier 1 solves for the optimal production quantity, maximizing his profit as given in Equation 4.6.

$$\pi_1(Q_1) = \alpha\, w Min(Q_1, H) + (1-\alpha) wL - \alpha k (H - Q_1)^+ + R_1(Q_1) - cQ_1. \qquad (4.6)$$

In this chapter, we use two benchmarks. The first benchmark is Supplier 1's production quantity when there is no spot market. With no spot market, $R_1=0$, and Supplier 1 produces L units if $c > \alpha(w+k)$ and H units otherwise. Hence, when Supplier 1 uses the spot market, $c > \alpha(w+k)$ is a necessary condition for an equilibrium in which $Q_1 \le H$. The second benchmark is the production quantity equilibrium and profits when there are two type 2 suppliers (no forward contract) in the industry, as given in Lemma 4.1.

Lemma 4.1: *For an industry with two type 2 suppliers, the equilibrium production quantities, suppliers' expected profits and expected spot market price are given by:*

$$Q_1^{E2} = \frac{B-c}{3} \qquad Q_2^{E2} = \frac{B-c}{3}, \quad \pi_1^{E2} = \frac{(B-c)^2}{9} \qquad \pi_2^{E2} = \frac{(B-c)^2}{9}, \quad E[p_{MP}]^{E2} = \frac{B+2c}{3}.$$

Proof. Using Equation 4.3, the equilibrium is given by the intersection of the following two best response functions: $Q_1(Q_2) = 0.5(B - Q_2 - c)$ and $Q_2(Q_1) = 0.5(B - Q_1 - c)$ ■

[7] The equality holds for β>-2.

Table 4.2 presents all feasible equilibria, when there is one supplier of each type, and the contract parameters, *w* and *k*, are such that Supplier 1 gives priority to contracted demand. The equilibrium in which $Q_1=L$ is not presented in Table 4.2, since then Supplier 1 never participates in the spot market.

Equilibrium	**The equilibrium values of** Q_1, Q_2, $E[q_1]$	**Conditions for existence**
T1: $Q_1 > H$ $Q_1 - L < \frac{B_L - Q_2}{2}$	$Q_1 = E[D] + (B-c)/3$ $Q_2 = (B-c)/3$ $E[q_1] = (B-c)/3$	$B \ge c + 3(1-\alpha)(H-L)$ $c > \alpha(H-L)(2+\beta)$ $w+k > c+(1-\alpha)(2+\beta)(H-L)$
T2: $Q_1 > H$ $Q_1 - L \ge \frac{B_L - Q_2}{2}$	$Q_1 = H + \frac{1}{2}\left(B_H - \frac{B-c}{3} - \frac{c}{\alpha}\right)$ $Q_2 = (B-c)/3$ $E[q_1] = (B-c)/3$	$c < \alpha(3B_H - B)/(3-\alpha)$ $c \le \alpha(2+\beta)(H-L)$ $\alpha(w+k) > c$
T3: $Q_1 = H$ $Q_1 - L < \frac{B_L - Q_2}{2}$	$Q_1 = H$ $Q_2 = \frac{B-c-(1-\alpha)(H-L)}{2}$ $E[q_1] = (1-\alpha)(H-L)$	$B \le c + 3(1-\alpha)(H-L)$ $w+k >$ $\frac{c(1+\alpha)-(1-\alpha)(B-(3+\alpha+2\alpha\beta)(H-L))}{2\alpha}$ $B > -c + (3+\alpha+2\alpha\beta)(H-L)$
T4: $Q_1 = H$ $Q_1 - L \ge \frac{B_L - Q_2}{2}$	$Q_1 = H$ $Q_2 = \frac{B(1+\alpha)-2c+\beta(1-\alpha)\alpha(H-L)}{3+\alpha}$ $E[q_1] = \frac{(1-\alpha)(B_L+c-\alpha\beta(H-L))}{3+\alpha}$	$\alpha(w+k) > c$ $c \ge \alpha(3B_H - B)/(3-\alpha)$ $w+k > \frac{3B_H - B + 2c}{3+\alpha}$
Liquidation Equilibrium: $L < Q_1 < H$ $Q_1 - L < \frac{B_L - Q_2}{2}$	$Q_1 = Q_1^0 - \beta\frac{2\alpha(H-L)}{3+\alpha}$ $Q_2 = Q_2^0 + \beta\frac{(1-\alpha)\alpha(H-L)}{3+\alpha}$ $Q_1^0 = L + \frac{2\alpha(w+k)-2c+(1-\alpha)(B+c)}{(1-\alpha)(3+\alpha)}$ $Q_2^0 = \frac{B(1+\alpha)-c-\alpha(k+w)}{3+\alpha}$ $E[q_1] = (1-\alpha)(Q_1 - L)$	$2\alpha(w+k)-2c+(1-\alpha)(B+c) >$ $2(1-\alpha)\beta(E[D]-L)$ $2\alpha(w+k)-2c+(1-\alpha)(B+c) <$ $(1-\alpha)(H-L)(3+\alpha+2\beta\alpha)$ $c > \alpha(w+k)$ $k+w > \frac{2B+c}{3} + \beta(1-\alpha)(H-L)$

Table 4.2: Production quantity equilibria in which Supplier 1 is "Loyal" to his loyal customers, when β>-2.

The analysis for deriving the equilibria in Table 4.2, and showing that there is no other equilibrium that satisfies Condition 4.4 when β>-2, is in Section 4.7. Examining Table 4.2, we learn that in equilibrium in which $Q_1>H$, the expected spot market supply is the same as when we have two type 2 suppliers in the market, and Supplier 1's contracts do not affect Supplier 2's behavior. In this chapter, we focus on the single equilibrium in which $L<Q_l<H$, and give a brief discussion of the intuition for the other four equilibria in Section 4.5. We name the equilibrium in which $L<Q_l<H$, the *Liquidation Equilibrium.* In this equilibrium, when contracted demand is low Supplier 1 sells his entire remaining inventory, (Q_1-L), on the spot market. When contracted demand is high, he sells his entire inventory to his loyal customers. The Liquidation Equilibrium is the only equilibrium in which Supplier 1 incurs a penalty cost when contracted demand is high, because his inventory is not sufficient to satisfy a high level of contracted demand. In addition, it is the only equilibrium in which the contracted unit price affects production quantities. The Liquidation Equilibrium prevails when (w+k) is high enough to make Supplier 1 give priority to demand originating from his loyal customers, but not high enough to make him produce H units, and, in addition, the expected spot market demand is such that Supplier 1 does not use the spot market as an additional channel. He uses it only when contracted demand is low. We find this equilibrium to be of most interest and we believe it captures the current situation of the majority of online marketplaces which position themselves as liquidation venues for overstocks. Notice that this is the only equilibrium in which both production quantities are a function of β.

In the following, we first analyze the Liquidation Equilibrium when the spot market demand is independent of contracted demand, i.e. when β = 0. Then, we analyze the affect of correlation between the demands on the results.

4.3 Analysis of the Liquidation Equilibrium with Independent Demands

Here we analyze the Liquidation Equilibrium to see how the activities in the contracts market and spot market interact when there is no correlation between the demands.

Lemma 4.2: *When spot market and contracted demands are independent, the following three conditions are necessary and sufficient for the existence of the Liquidation Equilibrium:*

A1. $c > \alpha(w+k)$

A2. $B < 1.5(k+w) - 0.5c$

A3. $(3+\alpha)(1-\alpha)(H-L) > 2\alpha(k+w) + B(1-\alpha) - c(1+\alpha)$

Proof: Using the results from Table 4.2, and $\beta=0$. Notice that when $B < 1.5(k+w) - 0.5c$ we don't need to require $2\alpha(k+w)+B(1-\alpha)-c(1+\alpha) > 0$ since

$2\alpha(k+w)+B(1-\alpha)-c(1+\alpha) = \alpha(1.5(k+w)-0.5c)+0.5\alpha(k+w-c)+B-c-B\alpha >$

$\alpha B+0.5\alpha(k+w-c)+B-c-B\alpha = B-c+0.5\alpha(k+w-c) > 0$.■

Condition A1 states that the production cost is large enough so that with no spot market Supplier 1 finds it optimal to produce L units (rather than H). Condition A2 ensures Supplier 1 does not offer some of his inventory on the spot market when contracted demand is high. Condition A3 ensures that Supplier 1 does not produce more than H units.

Based on Table 4.2, when the industry consists of one supplier with contracts (indexed as Supplier 1) and one with no contracts (indexed as Supplier 2) and conditions A1 to A3 are satisfied, the production quantities equilibrium is given by

$$Q_1^{E1} = L + \frac{(1-\alpha)(B+c) - 2c + 2\alpha(k+w)}{(3+\alpha)(1-\alpha)} \tag{4.7}$$

$$Q_2^{E1} = \frac{B(1+\alpha) - c - \alpha(k+w)}{(3+\alpha)}, \tag{4.8}$$

and Supplier 1 does not participate in the spot market when contracted demand is high, and offers his entire excess inventory on the spot market when contracted demand is low. Suppliers' expected profits are

$$\pi_1^{E1} = L(w-c) - \alpha k(H-L) + (1-\alpha)(Q_1^{E1} - L)^2 \tag{4.9}$$

$$\pi_2^{E1} = \frac{\left(B(1+\alpha) - \alpha(k+w) - c\right)^2}{(3+\alpha)^2} = (Q_2^{E1})^2 \tag{4.10}$$

and the expected spot market price is

$$E[p_{MP}]^{E1} = \frac{(1+\alpha)(B+c)+c-\alpha(k+w)}{3+\alpha}. \quad (4.11)$$

For the rest of this section we assume that the conditions specified in Lemma 4.2 hold.

4.3.1. Contracted Demand and Profits

We now consider how the contracted demand of Supplier 1 affects the production quantity and profit of Supplier 2, the supplier who works solely on the spot market. Equations 4.8 and 4.10 describe the equilibrium production quantity and profit of Supplier 2, given that Supplier 1 participates in the market with probability (1-α). When contracted demand and spot market demand are independent, Supplier 2's production lot and profit do not depend on the specific values of *H* and *L*, but only on the value of α. Hence, if expected contracted demand increases due to an increase in *H*, there is no effect on Supplier 2's equilibrium production lot and profit. However, if expected contracted demand increases due to an increase in α, then Supplier 2's production lot and profit do change.

From the perspective of Supplier 2 an increase in α (in the range of values bounded by conditions in Lemma 4.2) has two opposing effects. The probability that Supplier 1 participates in the spot market decreases, which is clearly to the benefit of Supplier 2. However, at the same time, when α increases it might also mean that Supplier 1 produces more, because he faces a higher probability of high contracted demand. Therefore, the combined effect on Supplier 2's profit depends on whether the expected lot offered on the market by Supplier 1, $(1-\alpha)(Q_1 - L) = \frac{(1-\alpha)(B+c)-2c+2\alpha(k+w)}{(3+\alpha)}$, is increasing or decreasing in α.

Proposition 4.1: *In the Liquidation Equilibrium, the production lot size and profit of Supplier 2 increase as the probability that Supplier 1 participates in the market increases.*

Proof: $\frac{\partial Q_2^{E1}}{\partial \alpha} = \frac{(B-k-w)(3+\alpha)-B(1+\alpha)+c+\alpha(k+w)}{(3+\alpha)^2} < 0$ if and only if $2B+c < 3(k+w)$, that is if and only if condition A2 holds.

$\pi_2^{E1} = (Q_2^{E1})^2$ and so $\frac{\partial \pi_2^{E1}}{\partial \alpha} = 2Q_2^{E1}\frac{\partial Q_2^{E1}}{\partial \alpha} < 0$ if and only if $2B + c < 3(k+w)$ and $Q_2^{E1} > 0$, which is assumed.■

Based on Proposition 4.1, an increase in the expected contracted demand that is caused by an increase in α has a negative affect on Supplier 2's profit. As α increases, though the probability that Supplier 1 participates in the spot market decreases, the quantity he produces and the expected quantity he offers on the spot market increase. Next, we consider which supplier benefits more from the existence of the spot market- the one who also has contracts, or the one that works solely on the spot market. Is Supplier 1's increase in profit (compared to his profit with no spot market) higher than Supplier 2's profit from the spot market?

Proposition 4.2: *Supplier 1's expected increase in profit due to the spot market is higher than Supplier 2's expected profit from the spot market.*

Proof: See Section 4.7.

Proposition 4.2 shows that in the Liquidation Equilibrium, a supplier with contracts actually profits from the existence of the spot market more than a supplier who produces only for the spot market. In addition, the type 2 supplier becomes worse off as the probability that he is a monopolist in the spot market increases. These two results are exhibited in Figure 4.1.

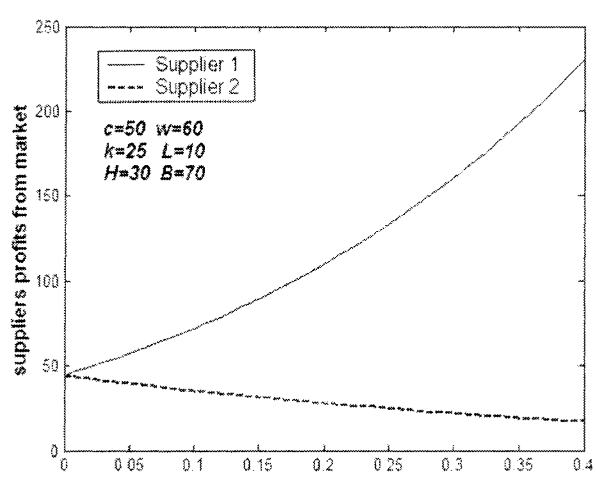

Figure 4.1: Supplier 2's profit and Supplier 1's profit increase from the spot market, as a function of α.

In Figure 4.1, the horizontal axis is truncated at α=0.4 because for larger values of α the conditions of Lemma 4.2 are not satisfied and there is a different equilibrium.

4.3.2 Investments in the Marketplace

In this section we find which of the two suppliers has a higher incentive to invest in increasing the demand on the spot market. Proposition 4.2 states that Supplier 1 benefits more from the existence of the spot market, however Proposition 4.3 shows that Supplier 2 (the supplier with no contracts) might benefit more than Supplier 1 from a marginal increase in the spot market demand, and so might have a higher incentive to invest in making the spot market more prevalent.

Proposition 4.3: *Both suppliers benefit from an increase in B, and the increase in Supplier 2's profit from an increase in B is larger than the increase in Supplier 1's profit iff B > (k+w).*

Proof: We can rewrite Supplier 1's profit as:

$\pi_1 = (Q_1^{E1} - L)^2(1-\alpha) + L(w-c) - \alpha k(H-L)$. Then notice that $(Q_1^{E1} - L)$ is increasing in B (Equation 4.8) and that $Q_1^{E1} \geq L$. Therefore, π_1 is increasing in B. Similarly, $\pi_2 = (Q_2^{E1})^2$ and Q_2^{E1} is increasing in B (Equation 4.9) and assumed non-negative. Therefore, π_2 is increasing in B. Taking the derivatives of profits with respect to B we find that $d\pi_2/dB > d\pi_1/dB$ iff $B > (k+w)$. ■

According to Proposition 4.3, when the spot market is small (low B) the type 1 supplier benefits more from an increase in spot market demand. When the market reaches a critical mass ($B > k+w$), the type 2 supplier benefits more from an increase in the demand on the spot market. Hence, suppliers with contracts would be the first to invest in new spot markets, while suppliers that do not have contracts are more likely to invest at a later stage, when the marketplace is more established.

4.3.3 Spot Markets and Prevalence of Contracting

We now consider how the spot market might affect long term relationships (contracts) between suppliers and their customers. Other papers have pointed out that as spot markets or secondary markets become more prevalent, forward contracting will become obsolete. Wu et al (2003) show that a seller who enjoys perfect market access will find no reason to contract. Tunca (2002) shows that if the exchange is sufficiently liquid, parties choose not to engage in contracting at all. However, these papers assume that spot market demand and contracted demand originate from the same buyer(s) and so, an increase in the spot market demand comes at the expense of contracted demand (demands are negatively correlated).

In the Liquidation Equilibrium, we consider markets in which $c > \alpha(k+w)$ (see Lemma 4.2). Hence, with no spot market, Supplier 1 engages in contracts and produces L units when his expected profit, $L(w-c)-\alpha k(H-L)$, is non-negative, and does not contract otherwise. With access to the spot market, his profit is $\pi_1^{E2} = (B-c)^2/9$ if he does not contract, and given by Equation 4.9 if he does. In this Section we denote by π_i^{E1} the profit of supplier i in the Liquidation Equilibrium with one supplier of each type, by π_i^{E2} the profit of supplier i in equilibrium when both suppliers are type 2, and by π_1^L supplier 1' profit when there is no spot market.

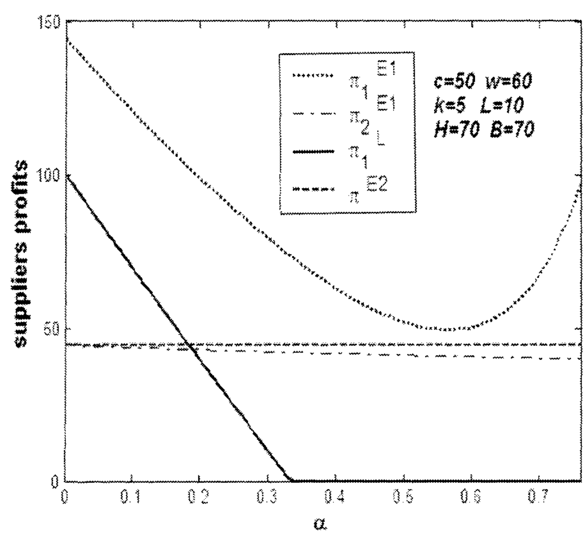

Figure 4.2: Suppliers profits as a function of α, when w=\$60, k=\$5 and c=\$50

If Supplier 1's expected profit from contracts with no spot market, given by $\pi_1^L = Max(0, L(w-c)-\alpha k(H-L))$, is zero and $\pi_1^{E1} - \pi_1^{E2} > 0$, it is not profitable for him to engage in forward contracts with fixed unit price w when there is no spot market. However, with access to the spot market, he finds it optimal to contract. The intuition behind this result is clear-- with no spot market the salvage value for unsold units is zero. But, with access to the spot market, the positive (though decreasing in quantity) salvage value reduces the risk of overestimating contracted demand, hence making contracting more appealing. This situation is exhibited in Figure 4.2: when $0.33 = \frac{L(w-c)}{k(H-L)} < \alpha < c/(k+w) = 0.77$ we have $\pi_1^L = 0$ and $\pi_1^{E1} - \pi_1^{E2} > 0$. But clearly, this is not always the case. In Figure 4.3 Supplier 1 prefers to use the spot market and not engage in contracts. That is $\pi_1^{E2} - \pi_1^{E1} > 0$, for a wide range of α values for which $\pi_1^L = 0$.

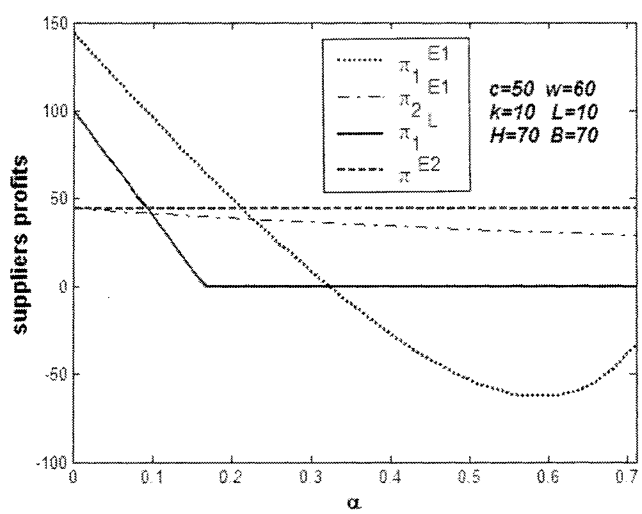

Figure 4.3: Suppliers profits as a function of α, when *w*=\$60, *k*=\$10 and *c*=\$50.

Hence, as much as buyers would like to set lower values of *w* and higher values of *k* in order to reduce procurement cost and insure supply, they must be careful, since if *w* decreases and *k* increases, Supplier 1 might choose not to contract, even when he can liquidate inventory on the spot market.

Notice that based on Proposition 4.2, $\pi_1^L > 0$ is sufficient for $\pi_1^{E1} > \pi_2^{E1}$. Hence, if contracting takes place when there is no spot market, Supplier 1's profit from contracts and spot market is higher than Supplier 2's profit. However, if in addition $\pi_1^{E2} > \pi_1^{E1}$, Supplier 1 eliminates contracting when using the spot market. As stated in the next proposition, elimination of contracting always increases Supplier 2's profit.

Proposition 4.4: *Eliminating contracting by Supplier 1 increases Supplier 2's profit.*

Proof: See Section 4.7.

4.4 Correlation between Demands

In this section we analyze the effect of correlation between spot market demand and contracted demand on the results from Section 4.3, and on the industry supply and spot market supply. In our model, the correlation coefficient has the same sign as β, because

$COV(D, d) = E[(D-E[D])(d-E[d])] = \alpha((H-E[D])\beta(H-E[D]))+(1-\alpha)((L-E[D])\beta(L-E[D]))$
$= \alpha(1-\alpha)(H-L)\,\beta(H-L) = \beta VAR[D]$.

In industries where contracted demand and spot market demand originate from two independent groups of buyers, but all buyers face the same end-customers demand, a high (low) level of contracted demand increases (decreases) the expected spot market demand. On the other hand, in industries where contracted demand increases at the expense of spot market demand, for example because buyers optimize on using both channels, a high (low) level of contracted demand decreases (increases) the expected spot market demand. Since both arguments are compelling, depending on the circumstances and the industry examined, we first examined the Liquidation Equilibrium assuming the demands are independent of each other. In this section we generalize all four propositions to cases in which spot market demand and contracted demand are correlated. The proofs are in Section 4.7.

Generalization of Proposition 4.1: *The production lot and profit of Supplier 2 increase as the probability that Supplier 1 participates in the market increases iff $\beta \geq 0$ or*
$\beta(3-\alpha^2-6\alpha)(H-L) < 3(k+w)-2B-c$.

Surprisingly, the result still holds when the demands on the two markets are positively correlated. When demands are positively correlated, we expect a decrease in α (which increases the probability Supplier 1 participates in the spot market) to make the spot market more attractive for Supplier 1, because $B_L = B-\beta\alpha(H-L)$ increases. That is, we expect Q_1 to increase and Supplier 2's profit to decrease as α decreases. The explanation for the counter intuitive result is the nature of the Liquidation Equilibrium. Supplier 1 uses the spot market mainly to reduce his penalty when contracted demand is high. Therefore, if the probability of high contracted demand decreases, there is less incentive to produce more than L units, and the decrease in $(Q_1\text{-}L)$ affects E[q_1] more than the increase in the participation probability, $(1\text{-}\alpha)$.

If demands are negatively correlated, a decrease in α reduces the expected spot market price when contracted demand is low. Hence, we expect the spot market to be less appealing for Supplier 1 as α decreases. A decrease in α also makes the spot market more appealing for Supplier 2 when contracted demand is high. The combined effect is complicated and depends on the relative values of the different parameters. However, notice that when $\alpha < 2\sqrt{3}-3$ and $3(k+w)\text{-}2B\text{-}c>0$ the result holds for every $\beta < 0$ (assuming it satisfies the conditions for existence of Liquidation Equilibrium listed in Table 4.2). Hence, if the Liquidation Equilibrium is feasible when demands are independent of each other, and the probability of high contracted demand is bounded by $2\sqrt{3}-3$, we can generalize Proposition 1 for every β.

Generalization of Proposition 4.2: *In the Liquidation Equilibrium, Supplier 1 benefits from the existence of the spot market more than Supplier 2, whether spot market demand and contracted demand are positively or negatively correlated.*

Generalization of Proposition 4.3: *In the Liquidation Equilibrium, a marginal increase in B increases both suppliers profits, and the increase in Supplier 2's profit is larger if and only if* $B > k+w-\beta(1-\alpha)(H-L)$.

Suppliers' profits are increasing and convex in B, and when spot market demand reaches a critical mass, Supplier 2 has a higher incentive than Supplier 1 to invest in extending the spot market. When demands on the two markets are positively correlated, the critical mass is smaller than when demands are negatively correlated. That is, in industries where demands on the two channels are positively correlated, we expect suppliers who do not have forward contracts to start investing in spot markets earlier.

Generalization of Proposition 4.4: *Eliminating contracting by Supplier 1 improves Supplier 2's condition if and only if* $\beta \geq 0$ *or* $2B + c < 3(k+w) - \beta(3-\alpha^2-6\alpha)(H-L)$.

In industries where the demands are positively correlated, elimination of contracting by Supplier 1 increases Supplier 2's profit. The reasoning is the same as the one offered for the generalization of Proposition 4.1.

When spot market demand and contracted demand are not independent of each other, the production quantities change linearly with the correlation parameter β (see Table 4.2). As β increases, Supplier 1 produces less because he expects a lower spot price (smaller B_L) in case he needs to use the spot market. Supplier 2, on the other hand, produces more, since he expects higher spot prices when *D=H*, and less competition when *D=L*. Since Supplier 1 uses the spot market only when contracted demand is low, he does not benefit from the increase in the expected spot price when contracted demand is high. Hence, as β increases, the spot market becomes less attractive for Supplier 1, but more attractive for Supplier 2.

When demands are negatively correlated, low contracted demand increases the expected spot market demand, and hence, the spot market channel compliments the contracts channel, and Supplier 1 can use it to manage risk. On the other hand, Supplier 2 is worse off (with negative correlation) and he produces less. We conclude that positive correlation between the demands favors suppliers that work solely on the spot market, while negative correlation favors suppliers that have contracts (they can better manage risk).

Proposition 4.5: *Everything else being equal, the supplier with contracts profits more in industries where demands are negatively correlated, while the supplier with no contracts profits more when demands are positively correlated.*

Next we examine the total industry supply and spot market supply.

Proposition 4.6: *Total industry supply and the spot market supply are decreasing in* β*.*

Proof:

$$\Delta Q = Q_1(\beta) + Q_2(\beta) - Q_1^0 - Q_2^0 = -\beta \frac{(1+\alpha)(E[D]-L)}{3+\alpha}$$

$$\Delta(\text{market supply}) = \text{E}[\text{q}_1] + Q_2(\beta) - E[q_1^0] - Q_2^0 = -\beta \frac{(1-\alpha)(E[D]-L)}{3+\alpha}$$

Corollary 4.7: *Everything else being equal, in industries where contracted demand and spot market demand are positively correlated, industry supply and spot market supply are smaller.*

According to Proposition 4.6, positive correlation is likely to make buyers worse off, while negative correlation increases industry supply and generally improves buyers' welfare. This result supports previous work that show an increase in buyers' surplus when they use the two channels, contracts and spot markets, to mange risk.

4.5 Discussion of Other Equilibria

In this Section we shortly analyze other equilibria presented in Table 4.2, and identify similarities and differences with the Liquidation Equilibrium.

Equilibria in which $Q_1 > H$

In the first two equilibria in Table 4.2, denoted by T1 and T2, expected spot market demand is high enough and production cost is low enough so that Supplier 1 produces more than H units and always participates in the spot market. In the first equilibrium, T1, Supplier 1 produces more than H units but less than $L + 0.5(B_L - Q_2)$ units, and so when contracted demand is low, he offers his entire excess inventory on the spot market. In the second equilibrium, T2, Supplier 1 produces more than $L + 0.5(B_L - Q_2)$ units and so when contracted demand is low, he offers only part of his excess inventory on the spot market. Since Supplier 2's production quantity is the same in both cases, Supplier 1 produces more in Equilibrium T2 than in Equilibrium T1. The Equilibrium production quantities and profits for T1 and T2 are given by:

$$Q_1^{T1} = E[D] + (B - c)/3$$

$$Q_1^{T2} = H + 0.5(B_H - (B - c)/3 - c/\alpha)$$

$$Q_2^{T1} = Q_2^{T2} = (B - c)/3$$

$$\pi_1^{T1} = (B - c)^2/9 + E[D](w - c) - (1 + \beta)\alpha(1 - \alpha)(H - L)^2$$

$$\pi_1^{T2} = (B - c)^2/9 + E[D]w - cH + (1 - \alpha)(c - \alpha\beta(H - L))^2/(4\alpha)$$

$$\pi_2^{T1} = \pi_2^{T2} = (B - c)^2/9$$

Supplier 2's production quantity and profit, in both T1 and T2, are not affected by Supplier 1's forward contracts; Supplier 2 produces the same quantity as when competing with another type 2 supplier. In addition, in both T1 and T2, the expected spot market supply is the same as when there are no forward contracts in the industry.

Similarly to our result for the Liquidation Equilibrium, in both T1 and T2, Supplier 1 benefits from the spot market more than Supplier 2. In equilibrium T1 Supplier 1 benefits from the spot market more than Supplier 2 because for this equilibrium to exist we must have $w+k > c+(1-\alpha)(2+\beta)(H-L)$ and $c>\alpha(2+\beta)(H-L)$, and therefore

$\pi_1^{T1} - (Lw - cL - \alpha k(H-L)) - \pi_2^{T1} = \alpha(H-L)(k+w-c-(1+\beta)(1-\alpha)(H-L)) > 0$ and

$\pi_1^{T1} - (E[D]w - cH) - \pi_2^{T1} = (1-\alpha)(H-L)(c-(1+\beta)\alpha(H-L)) > 0$.

In equilibrium T2, Supplier 1 benefits from the spot market more than Supplier 2 since

$$\pi_1^{T2} - (E[D]w - cH) - \pi_2^{T1} = (1-\alpha)(c-\beta\alpha(H-L))^2 > 0.$$

For equilibrium T2 to be feasible it must be that $\alpha(w+k)>c$, while this condition is not necessary for equilibrium T1. That is, if without the spot market it is optimal for Supplier 1 to produce only L units, then T1 might be feasible, but T2 is not. Hence, we expect T1 to resemble the Liquidation Equilibrium more than T2. Indeed, as in the Liquidation Equilibrium, in T1 Supplier 1 benefits from negative correlation between spot market demand and contracted demand.

Equilibria in which Q_1=H

In the third and fourth equilibria in Table 4.2, denoted by T3 and T4, the value of $(w+k)$ is high enough, but the spot market is not attractive enough, and so Supplier 1 produces exactly H units. He does not risk not fully satisfying loyal customers' demand, but he is not using the spot market as an additional channel (i.e. he uses it only when contracted demand is less than his inventory).

In Equilibrium T4, Supplier 1 uses the spot market when contracted demand is low, to sell only part of his excess inventory, while in T3 Supplier 1 sells his entire access inventory. Hence, we can conclude that Q_2 is smaller in T3.

A necessary condition for T4 to be feasible is that with no spot market Supplier 1 produces H units. This condition is not necessary for T3. Therefore, the two main differences between T4 and T3 are whether $\alpha(w+k) > c$ is a necessary condition, and whether Supplier 1 offers his entire access inventory on the spot market when contracted demand is low.

Notice the similarity between Supplier 2's production quantity in T3 and his production quantity in the Liquidation Equilibrium:

$$Q_2^{T3} = \frac{B(1+\alpha)-2c}{3+\alpha} + \frac{\beta(1-\alpha)(E[D]-L)}{3+\alpha}$$

$$Q_2^{LE} = \frac{B(1+\alpha)-c-\alpha(k+w)}{3+\alpha} + \frac{\beta(1-\alpha)(E[D]-L)}{3+\alpha}.$$

Changes in β, E[D] and B have the same effect on both quantities. However, while in the Liquidation Equilibrium Supplier 2's production quantity depends on (w+k), in both T3 and T4 it is independent of (w+k). Studying these two equilibria can be an interesting extension to this work.

4.6 Conclusions

In this chapter we focus on one equilibrium in which the supplier with contracts sells his entire excess inventory on the spot market when contracted demand is low, and does not participate in the spot-market when contracted demand is high. This equilibrium matches popular practice in industry: suppliers with contracts give priority to demand originating from their contracts, and use spot markets only to liquidate excess inventory. The main reason for the prevalence of this practice might be the limited liquidity in spot markets.

The existence of spot markets enables suppliers who have forward contracts to salvage excess inventory, reducing the cost of overestimating demand from their loyal customers. Hence, having access to the spot market clearly improves the condition of these suppliers. Surprisingly, we show that in the Liquidation Equilibrium described above, suppliers who have contracts not only increase their profits due to the existence of the spot market but they benefit from it more than those suppliers who work solely on the spot market. In addition, when the spot market is small, the supplier that has contracts has a higher incentive to invest in extending the spot market. When the spot market exceeds a threshold size this situation is reversed and the supplier with no contracts benefits more from making the spot market more prevalent. We also show that the supplier who works solely on the spot market would be better off if the other supplier eliminates contracting, i.e. he prefers to compete with suppliers who work solely on the spot market than with suppliers who use the spot market for inventory liquidation. But, contracting can take place due to the existence of the spot market, while being unprofitable otherwise. Buyers who prefer forward contracting to waiting for the spot market can benefit from the existence of the spot market, as suppliers can accept contracts with lower unit price.

For the Liquidation Equilibrium we show that positive correlation between the spot market demand and the contracted demand favors suppliers that work solely on the spot market, while negative correlation favors suppliers that have contracts (they can better manage risk). In addition, total industry supply and the spot market supply are higher when demands are negatively correlated. Thus, positive correlation is likely to make buyers worse off, while negative correlation increases industry supply and generally improves buyers' welfare.

4.7 Technical Derivations and Proofs

Derivation of Table 4.2

Using Equations 4.5 and 4.6, we first write Supplier 1's profit as a function of the production quantities for the four cases, depending on whether $Q_1 \geq H$ and whether $Q_1 < L + 0.5(B_L - Q_2)$.

1. If $Q_1 \geq \text{H}$ and $Q_1 < L + 0.5(B_L - Q_2)$

$$\pi_1 = wED - cQ_1 + \alpha(Q_1 - H)(B_H - Q_2 - (Q_1 - H)) + (1-\alpha)(Q_1 - L)(B_L - Q_2 - (Q_1 - L))$$

$$\frac{\partial \pi_1}{\partial Q_1} = -c + B - Q_2 - 2\alpha(Q_1 - H) - 2(1-\alpha)(Q_1 - L) = -c + B - Q_2 - 2(Q_1 - E[D])$$

2. If $Q_1 \geq \text{H}$ and $Q_1 \geq L + 0.5(B_L - Q_2)$

$$\pi_1 = wED - cQ_1 + \alpha(Q_1 - H)(B_H - Q_2 - (Q_1 - H)) + 0.25(1-\alpha)(B_L - Q_2)^2$$

$$\frac{\partial \pi_1}{\partial Q_1} = -c + \alpha(B_H - Q_2 - 2(Q_1 - H))$$

3. If $L \leq Q_1 < \text{H}$ and $Q_1 < L + 0.5(B_L - Q_2)$

$$\pi_1 = \alpha w Q_1 + (1-\alpha)wL - cQ_1 - \alpha k(H - Q_1) + (1-\alpha)(Q_1 - L)(B_L - Q_2 - (Q_1 - L))$$

$$\frac{\partial \pi_1}{\partial Q_1} = \alpha(w + k) - c + (1-\alpha)(B_L - Q_2 - 2(Q_1 - L))$$

4. $L \leq Q_1 < \text{H}$ and $Q_1 \geq L + 0.5(B_L - Q_2)$

$$\pi_1 = \alpha w Q_1 + (1-\alpha)wL - cQ_1 - \alpha k(H - Q_1) + 0.25(1-\alpha)(B_L - Q_2)^2$$

$$\frac{\partial \pi_1}{\partial Q_1} = \alpha(w+k) - c$$

We search for production quantity equilibrium in each of the above four cases. We use Equation 4.3, to determine Supplier 2's best response function. In addition, we use the following relationships:

$$B_H = B + \beta(H - ED) = B + \beta(1-\alpha)(H-L)$$
$$B_L = B - \beta(ED - L) = B - \beta\alpha(H-L)$$
$$B_H - B_L = \beta(H-L)$$

1. Assume there is an equilibrium in which $H \le Q_1 < L + \frac{B_L - Q_2}{2}$.

We first look for an interior solution. Substituting $E[q_1] = Q_1 - E[D]$, in Equation 4.3 we find Supplier 2's best response function: $Q_2(Q_1) = (B - c - Q_1 + E[D])/2$. Supplier 1's best response function is given by: $Q_1(Q_2) = (B - c - Q_2 + 2E[D])/2$. Solving simultaneously the two best response functions for production quantities we get $Q_1 = E[D] + (B-c)/3$ and $Q_2 = (B-c)/3$. The Equilibrium profits are $\pi_1 = (B-c)^2/9 + E[D](w-c) - (1+\beta)\alpha(1-\alpha)(H-L)^2$ and $\pi_2 = (B-c)^2/9$.

This equilibrium is feasible and satisfies condition 4.4 if and only if the following conditions hold:

1.1 $Q_1 > H$ iff $B > c+3(1-\alpha)(H-L)$.

1.2 $Q_1 - L < 0.5(B_L - Q_2)$ iff $c > (2+\beta)\alpha(H-L)$

1.3 $w+k > B_H - Q_2 - 2(Q_1 - H)$ iff $w+k > c + (1-\alpha)(2+\beta)(H-L)$.

Notice that B_H-Q_2-2(Q_1-H) > B_L-Q_2-2(Q_1-L) because we assumed β>-2. Also, the expected spot price is non-negative, for each realization of contracted demand:

$$E\left[p_{MP} \middle| D = H\right] = (B + 2c + 3(1-\alpha)(1+\beta)(H-L))/3 > 0.$$

$$E\left[p_{MP} \middle| D = L\right] = (B + 2c - 3\alpha(1+\beta)(H-L))/3 >$$
$$(3(1-\alpha)(H-L) + 3\alpha(H-L)(2+\beta) - 3\alpha(1+\beta)(H-L))/3 = (H-L) > 0,$$

where the first inequality is based on conditions 1.1 and 1.2.

This equilibrium is denoted by T1 and is listed in Table 4.2.

Next we search for equilibrium with $Q_1=H$ and $Q_1 < L + 0.5(B_L - Q_2)$. $E[q_1] = (1-\alpha)(H-L)$ and using Equation 4.3 we find Supplier 2's production quantity to be $Q_2 = 0.5(B-(1-\alpha)(H-L)-c)$. For this to be an Equilibrium (i.e. for Supplier 1 not to deviate), Supplier 1's profit must be not increasing in Q_1 when $Q_1 \geq H$, that is $\frac{\partial \pi_1}{\partial Q_1}\Big|_{Q_1=H} = -c + B - Q_2 - 2(1-\alpha)(H-L) \leq 0$, and not decreasing in Q_1 when $Q_1 < H$, that is $\frac{\partial \pi_1}{\partial Q_1}\Big|_{Q_1=H} = \alpha(w+k) - c + (1-\alpha)(B_L - Q_2 - 2(H-L)) \geq 0$. In addition we need to require $Q_1=H$ and $Q_2 = 0.5(B-(1-\alpha)(H-L)-c)$ to satisfy $H < L + \frac{B_L - Q_2}{2}$, and $(w+k) > B_H - Q_2$. To summarize, this equilibrium is feasible if and only if:

1.4 $B \leq c+3(1-\alpha)(H-L)$

1.5 $w+k > \frac{c(1+\alpha)-(1-\alpha)(B-(3+\alpha+2\alpha\beta)(H-L))}{2\alpha}$

1.6 $B > -c+(3+\alpha+2\alpha\beta)(H-L)$

1.7 $w+k > 0.5(B+c+(1-\alpha)(1+2\beta)(H-L))$

But $0.5(B+c+(1-\alpha)(1+2\beta)(H-L)) > \frac{c(1+\alpha)-(1-\alpha)(B-(3+\alpha+2\alpha\beta)(H-L))}{2\alpha}$ if and only if $B > c+3(1-\alpha)(H-L)$. Hence, condition 1.7 is not necessary.

Notice that B_H-Q_2-$2(Q_1$-$H)$ > B_L-Q_2-$2(Q_1$-$L)$ because we assumed β>-2. Also, the expected spot price is nonnegative, for each realization of contracted demand:

$E[p_{MP} | D = H] = 0.5(B+c+(1-\alpha)(1+2\beta)(H-L)) > 0.5(B+c-(1-\alpha)(H-L) > 0$

since β>-1 and since $c \geq B+3(1-\alpha)(H-L)$ according to condition 1.4.

$E[p_{MP} | D = L] = 0.5(B+c-(1-\alpha+2\alpha\beta)(H-L)) >$

$0.5((3+\alpha+2\alpha\beta)(H-L)-(1-\alpha+2\alpha\beta)(H-L)) = 2(H-L) > 0$, where the first inequality holds using condition 1.6.

This equilibrium is denoted by T3 and is listed in Table 4.2.

2. Assume there is equilibrium in which $Q_1 \geq H$ and $Q_1 \geq L + \frac{B_L - Q_2}{2}$.

We first look for an interior solution. Substituting

$E[q_1] = \alpha(Q_1 - H) + (1-\alpha)(B_L - Q_2)/2$ in Equation 4.3 we get Supplier 2's best response function: $Q_2(Q_1) = (B + \alpha B_H - 2c + 2\alpha(H - Q_1))/(3+\alpha)$. Supplier 1's best response function is given by $Q_1 = -c/(2\alpha) + 0.5(B_H - Q_2 + 2H)$. Solving the two best response functions simultaneously we get $Q_1 = H + 0.5(B_H - (B-c)/3) - c/(2\alpha)$ and $Q_2 = (B-c)/3$. The equilibrium profits are

$\pi_1 = (B-c)^2/9 + E[D]w - cH + (1-\alpha)(c - \beta\alpha(H-L))^2/(4\alpha)$ and $\pi_2 = (B-c)^2/9$.

This equilibrium is feasible and satisfies Condition 4.4, if and only if the following conditions hold:

2.1 $Q_1 > H$ if and only if $c(3-\alpha) < \alpha(2B + 3\beta(1-\alpha)(H-L)) = \alpha(3B_H - B)$.

2.2 $Q_1 - L \geq 0.5(B_L - Q_2)$ if and only if $c \leq \alpha(2+\beta)(H-L)$.

2.3 $w+k > B_H - Q_2 - 2(Q_1 - H)$ if and only if $\alpha(w+k) > c$.

Notice that $B_H - Q_2 - 2(Q_1 - H) > B_L - Q_2 - 2(Q_1 - L)$ because assumed $\beta > -2$. This equilibrium is denoted by T2 an listed in Table 4.2.

We next search for equilibrium in which $Q_1 = H$ and $Q_1 \geq L + \frac{B_L - Q_2}{2}$. First, $c < \alpha(w+k)$ is necessary, or else Supplier 1 deviates and produces less than H. For Supplier 1 not to deviate and produce more then H units we must have that the derivative of his profit when $Q_1 \to H$ from above, $\left.\frac{\partial \pi_1}{\partial Q_1}\right|_{Q_1 = H} = \alpha(B_H - Q_2) - c$, is not positive.

Substituting $E[q_1] = (1-\alpha)(B_L - Q_2)/2$ in Equation 4.3 we find that Supplier 2's production quantity is $Q_2 = \frac{B + \alpha B_H - 2c}{3+\alpha}$. This equilibrium is feasible if and only if:

2.4 $c < \alpha(w+k)$.

2.5 $c \geq \alpha(B_H - Q_2)$ if and only if $(3-\alpha)c \geq \alpha(3B_H - B)$.

2.6 $w+k > B_H - Q_2$ which holds if and only if $w + k > \frac{3B_H - B + 2c}{3+\alpha}$.

Notice that $B_H - Q_2 > B_L - Q_2 - 2(H-L)$ because assumed $\beta > -2$,

This equilibrium is denoted by T4 and is listed in Table 4.2.

3. Assume there is equilibrium in which $L \le Q_1 < \mathrm{H}$ and $Q_1 - L < \frac{B_L - Q_2}{2}$.

Substituting $E[q_1] = (1-\alpha)(Q_1 - L)$ in Equation 4.3 we get Supplier 2's best response function. Supplier 1's profit is given by

$\pi_1 = \alpha w Q_1 + (1-\alpha)wL - cQ_1 - \alpha k(H - Q_1) + (1-\alpha)(Q_1 - L)(B_L - Q_2 - (Q_1 - L))$.

Using FOC, we get Supplier 1's best response and then solving the two best response functions simultaneously we get $Q_1 = L + \frac{2\alpha(w+k) - 2c + (1-\alpha)(B+c)}{(1-\alpha)(3+\alpha)} - \beta\frac{2(ED-L)}{3+\alpha}$ and $Q_2 = \frac{B(1+\alpha) - c - \alpha(k+w)}{3+\alpha} + \beta\frac{(1-\alpha)(ED-L)}{3+\alpha}$. This equilibrium is feasible and satisfies Condition 4.4 if and only if the following conditions hold:

3.1 $L \le Q_1 < H$ which holds if and only if

$2\beta(1-\alpha)(E[D]-L) \le 2\alpha(w+k) - 2c + (1-\alpha)(B+c) < (1-\alpha)(H-L)(3+\alpha+2\alpha\beta)$

3.2 $Q_1 - L < \frac{B_L - Q_2}{2}$ which holds if and only if $c > \alpha(k+w)$

3.3 $w+k > B_H - Q_2$ which holds if and only if $w + k > \frac{2B+c}{3} + \beta(1-\alpha)(H-L)$.

$w+k > B_L - Q_2 - 2(Q_1 - L)$ holds if and only if $w + k > c$ which was assumed.

This equilibrium is named the Liquidation Equilibrium, and is listed in Table 4.2.

4. Assume there is equilibrium in which $L \le Q_1 < \mathrm{H}$ and $Q_1 - L \ge \frac{B_L - Q_2}{2}$.

$\pi_1 = \alpha w Q_1 + (1-\alpha)wL - cQ_1 - \alpha k(H - Q_1) + 0.25(1-\alpha)(B_L - Q_2)^2$. Hence, Supplier 1's profit is linearly increasing in Q_1 when $c < \alpha(w+k)$, and linearly decreasing in Q_1 otherwise, and an equilibrium with interior solution is not possible. We thus examine if equilibrium in which $Q_1 = L + (B_L - Q_2)/2$ is feasible.

In this case we must have $c > \alpha(w+k)$, or else Supplier 1 would deviate and produce more. For Supplier 1 not to produce less than $L + (B_L - Q_2)/2$ we must have that the derivative of his profit when $Q_1 \to L + (B_L - Q_2)/2$ from below is positive. Taking the derivative of

$\pi_1 = \alpha w Q_1 + (1-\alpha)wL - cQ_1 - \alpha k(H - Q_1) + (1-\alpha)(Q_1 - L)(B_L - Q_2 - (Q_1 - L))$

with respect to Q_1 we get $\alpha(w+k)-c+(1-\alpha)(B_L-Q_2-2(Q_1-L))$. Evaluating this derivative at

$Q_1 = L+(B_L-Q_2)/2$, we get $\alpha(w+k)-c<0$. Hence, for any value of Q_2, Supplier 1 will deviate and produce less than $L+(B_L-Q_2)/2$. Such equilibrium is not feasible. ■

Proof of Proposition 4.2

Using Equations 4.10 and 4.11, Supplier 1's expected profit increase from his access to the spot market is higher than Supplier 2's expected profit from the spot market if and only if

$$\frac{((1-\alpha)(B+c)-2c+2\alpha(k+w))^2}{(1-\alpha)(3+\alpha)^2}-\frac{(B(1+\alpha)-\alpha(k+w)-c)^2}{(3+\alpha)^2}>0 \text{ that is iff}$$

$$((1-\alpha)(B+c)-2c+2\alpha(k+w))^2-(1-a)(B(1+a)-a(k+w)-c)^2>0.$$

The two roots of the quadratic equation in B,

$$((1-\alpha)(B+c)-2c+2\alpha(k+w))^2-(1-a)(B(1+a)-a(k+w)-c)^2=0 \text{ are}$$

$$B_{1,2}=k+w\pm\frac{\sqrt{1-\alpha}}{1-\alpha}(k+w-c) \text{ and the second derivative of}$$

$((1-\alpha)(B+c)-2c+2\alpha(k+w))^2-(1-a)(B(1+a)-a(k+w)-c)^2$ with respect to B is $\alpha(\alpha+3)(\alpha-1)<0$, so that the expression is concave in B. Hence, the initial inequality holds only when

$$B\in\left(k+w-\frac{\sqrt{1-\alpha}}{1-\alpha}(k+w-c),k+w+\frac{\sqrt{1-\alpha}}{1-\alpha}(k+w-c)\right).$$

Remember that according to Lemma 4.2 and our assumption that $B>c$ we consider only cases in which $B\in(c,\ 1.5(k+w)-0.5c)$. We now show that this later range is contained within

$$B\in\left(k+w-\frac{\sqrt{1-\alpha}}{1-\alpha}(k+w-c),k+w+\frac{\sqrt{1-\alpha}}{1-\alpha}(k+w-c)\right).$$

$$k+w+\frac{\sqrt{1-\alpha}}{1-\alpha}(k+w-c)>k+w+0.5(k+w-c) \text{ because } \sqrt{1-\alpha}<2.$$

$$k+w-\frac{\sqrt{1-\alpha}}{1-\alpha}(k+w-c)<c \text{ iff } (k+w)\left(1-\frac{\sqrt{1-\alpha}}{1-\alpha}\right)<c\left(1-\frac{\sqrt{1-\alpha}}{1-\alpha}\right), \text{ that is iff}$$

$k+w > c$ (since $1/\sqrt{1-\alpha} > 1$ we switch the inequality direction) which clearly holds since $w>c$.

Hence, when conditions in Lemma 4.2 and $B > c$ hold, Supplier 1 always profits more than Supplier 2 from the existence of the spot market. ■

Proof of Proposition 4.4

Supplier 2's profit when Supplier 1 has contracts is given by:

$\pi_2^{E1} = (B - C + \alpha(B - k - w))^2 / (3+\alpha)^2$. Supplier 2's profit when Supplier 1 does not have contracts is $\pi_2^{E2} = (B-C)^2/9$. For $\alpha=0$ the two are the same and so if π_2^{E1} is increasing in α, Supplier 2 is better off when Supplier 1 has contracts, and if π_2^{E1} is decreasing in α Supplier 2 is better off when Supplier 1 eliminates contracting. We next show that given the conditions of Lemma 4.2, specifically given $B < 1.5(k+w) - 0.5c$, π_2^{E1} is decreasing in α.

$\frac{\partial \pi_2^{E1}}{\partial \alpha} = \frac{2(B-c+\alpha(B-k-w))(2B-3(k+w)+c)}{(3+\alpha)^3}$ and $2B-3(k+w)+c < 0$ since $B < 1.5(k+w)-0.5c$. In addition, $B-c+\alpha(B-k-w) > 0$ since otherwise $Q_2^{E1} < 0$ (see Equation 4.8). Hence, π_2^{E1} is decreasing in α and $\pi_2^{E1} < \pi_2^{E2}$ whenever the conditions of Lemma 4.2 hold.■

Generalization of Proposition 4.1

$$\frac{\partial Q_2}{\partial \alpha} = \frac{2B+c-3(k+w)}{(3+\alpha)^2} + \beta\frac{(-\alpha^2-6\alpha+3)(H-L)}{(3+\alpha)^2} < 0 \text{ if and only if}$$

$2B+c < 3(k+w) - \beta(3-\alpha^2-6\alpha)(H-L)$. But, for the Liquidation Equilibrium to exist it must be that $2B+c < 3(k+w) - 3\beta(1-\alpha)(H-L)$ (see Table 4.2). Since

$(3-\alpha^2-6\alpha) \le 3(1-\alpha)$ for every $\alpha \in [0,1]$ we conclude that for $\beta \ge 0$ we ahve $\frac{\partial Q_2}{\partial \alpha} < 0$.

When $\beta < 0$, $\frac{\partial Q_2}{\partial \alpha} < 0$ if and only if $2B+c < 3(k+w) - \beta(3-\alpha^2-6\alpha)(H-L)$ since this constraint is more binding. As before $\pi_2 = (Q_2)^2$ and, $\frac{\partial \pi_2}{\partial \alpha} = 2Q_2\frac{\partial Q_2}{\partial \alpha} < 0$ if and only if

$\frac{\partial Q_2}{\partial \alpha} < 0$. Hence, Proposition 4.1 holds with any β>0, which satisfies the conditions for the Liquidation Equilibrium listed in Table 4.2 , but only when

$2B + c < 3(k+w) - \beta(3 - \alpha^2 - 6\alpha)(H - L)$ for $\beta < 0$. ■

Generalization of Proposition 4.2

Supplier 1's expected profit increase from his access to the spot market is higher than Supplier 2's expected profit from the spot market if and only if

$(1-\alpha)(Q_1 - L)^2 - Q_2^2 > 0$ that is iff

$(1-\alpha)\left(Q_1^0 - \beta\frac{2}{3+\alpha}(E[D]-L) - L\right)^2 - \left(Q_2^0 + \beta\frac{(1-\alpha)}{3+\alpha}(E[D]-L)\right)^2 > 0$ Which holds for

$$B \in \begin{pmatrix} k+w-\beta(H-E[D]) - \frac{\sqrt{1-\alpha}}{1-\alpha}(k+w-c-\beta(H-E[D])), \\ k+w-\beta(H-E[D]) + \frac{\sqrt{1-\alpha}}{1-\alpha}(k+w-c-\beta(H-E[D])) \end{pmatrix}.$$

Notice that $k+w-c-\beta(H-E[D])$>0, since for the Liquidation Equilibrium to exist it must be that $k+w-\beta(H-E[D]) > 2B/3 + c/2$ (see Table 4.2) and $2B/3+c/3>c$ since $B>c$. According to Table 4.2 and our assumption that $B > c$, we consider only cases in which $B \in (c,\ 1.5(k+w) - 1.5\beta(H-E[D]) - 0.5c)$. We now show that this later range is contained within

$$B \in \begin{pmatrix} k+w-\beta(H-E[D]) - \frac{\sqrt{1-\alpha}}{1-\alpha}(k+w-c-\beta(H-E[D])), \\ k+w-\beta(H-E[D]) + \frac{\sqrt{1-\alpha}}{1-\alpha}(k+w-c-\beta(H-E[D])) \end{pmatrix}.$$

$$\begin{aligned} &k+w-\beta(H-E[D]) + \frac{\sqrt{1-\alpha}}{1-\alpha}(k+w-c-\beta(H-E[D]) > \\ &k+w-\beta(H-E[D]) + 0.5(k+w-c-\beta(H-E[D]) = \\ &1.5(k+w) - 0.5c - 1.5\beta(H-E[D]), \end{aligned}$$

where the inequality holds because $\sqrt{1-\alpha} < 2$.

$k+w-\beta(H-E[D]) - \frac{\sqrt{1-\alpha}}{1-\alpha}(k+w-c-\beta(H-E[D]) < c$ iff

$(k+w)\left(1-\frac{\sqrt{1-\alpha}}{1-\alpha}\right)<\left(1-\frac{\sqrt{1-\alpha}}{1-\alpha}\right)(c+\beta(H-E[D]))$, and Since $1/\sqrt{1-\alpha}>1$ the last inequality holds iff $(k+w)>c+\beta(H-E[D])$. But, according to Table 4.2, for the Liquidation Equilibrium to exist it must be that $(k+w)>(2B+c)/3+\beta(H-E[D])$, and $(2B+c)/3>c$ whenever $B>c$. Therefore, when conditions for existence of Liquidation Equilibrium, given in Table 4.2, and $B>c$ hold, Supplier 1 always profits more than Supplier 2 from the existence of the spot market.

Generalization of Proposition 4.3

We can rewrite suppliers profits as $\pi_1 = L(w-c)-\alpha k(H-L)+(1-\alpha)(Q_1-L)^2$ and $\pi_2 = Q_2^{\,2}$. Both suppliers' profits are increasing in B because Q_1^0 and Q_2^0 are increasing in B and the terms that depend on β are not functions of B. In addition $\partial\pi_2/\partial B > \partial\pi_1/\partial B$ if and only if $B>k+w-\beta(H\text{-E}[D])$ because

$\partial\pi_2/\partial B-\partial\pi_1/\partial B = 2\alpha(B-k-w+\beta(1-\alpha)(H-L))/(3+\alpha)$.

Generalization of Proposition 4.4

Supplier 2's profit when Supplier 1 has contracts is

$\pi_2=\left(\frac{(B-c+\alpha(B-k-w))+\beta(1-\alpha)\alpha(H-L)}{(3+\alpha)}\right)^2$. Supplier 2's profit when Supplier 1 does not have contracts is $\pi_2^{E2}=(B-c)^2/9$. For α=0 the two are the same and so if π_2 is increasing in α Supplier 2 is better off when Supplier 1 has contracts, and if π_2 is decreasing in α Supplier 2 is better off when Supplier 1 eliminates contracting. We already showed in the generalization of Proposition 4.1 that π_2 is decreasing in α if and only if $\beta\geq 0$ or $\beta<0$ and $2B+c<3(k+w)-\beta(3-\alpha^2-6\alpha)(H-L)$.■

Chapter 5

The Evolving Market for Independent Software Vendors Selling Specialized Business Applications

5.1. Introduction

Value-added resellers, VARs, are companies that make the product of another firm (the manufacturer/OEM) a key component of their product or service offerings, and in return the OEM provides them with technological support, training and sometimes even funds. This type of alliance prevails in the software industry: the main firm produces and sells base-software that is very general (not catered to specific customer segments), and therefore independent software vendors (ISVs) can enhance the value of the product to consumer segments by offering specialized applications and supplementary services. For example, Microsoft uses resellers to promote and sell Microsoft solutions and products, and to provide services for, and build applications on, the Microsoft platform.

> *Microsoft is creating a base layer of technology, including functions such as order processing and inventory management, upon which smaller software makers can build specialized applications (that cater to specific customer segments).* (*Business Week,* April 21, 2003)

In many cases, the base-software alone is not valuable for firms, without the addition of suitable business applications (examples are operating systems such as Windows and database management systems such as Oracle). Firms can either develop business applications in-house or, when available, purchase packaged applications from ISVs or from the base-software producer. In most cases, the balance of power lies with the base-software producer, who controls the degree of openness of the interfaces to the base-software and how much information about the base-software and its interfaces to disclose to the ISVs. Gawer and Cusumano (2002) explain the incentives of platform leaders (firms whose product is the foundation on which other companies build their products) to cooperate with firms selling complements:

> *Platform leaders might rest easier if they had the resources to create all possible complementary products themselves for every market around the world. But this is impossible... As a result, nearly all the platform leaders we observed have had to work closely with other firms to create initial applications and then new generations of complementary products. Platform leaders and complementary innovators have great incentives to cooperate, however, because their combined efforts can increase the potential size of the pie for everyone. Complements can draw new customers in, inducing them to buy the core product. The decision of what complements to make inside and what to leave to external firms is probably the single most important issue that platform leaders have to decide.* (Gawer and Cusumano, 2002)

A good example of the growing relationships between base-software producers and ISVs selling specialized applications, is Oracle. As of June 2004, Oracle had 200,000 database customers worldwide and 13,000 applications (Oracle's Applications) customers worldwide, so that the majority of the applications running on its DB are from third parties. Charles Phillips, President of Oracle Corporation, explained in Oracle's ISV forum in June 2004: "Most applications that are running in the marketplace today were not written by Oracle. Oracle's Application business is a very tiny percentage of what customers are doing with the Oracle database. Much broader impact in making it pervasive have been the ISVs – showcasing the capability of the product and taking advantage of it and bringing value to customers."

In this chapter we first examine why firms choose packaged applications rather than in-house development. We develop a conceptual and analytical model of the interaction between a base-software producer, ISVs selling specialized applications that run on the base-software, and user firms, in a horizontally differentiated market. The base-software can not be used by itself, and an application is required for the base-software to be valuable to a firm. Firms choose whether to develop a specialized application within the company, achieving perfect fit with the company needs at a high development cost, or buy a ready-to-use application, which might not exactly match their needs. The misfit cost from using the packaged application increases with the size (operation scale) of the firm. Hence, large firms have a higher misfit cost than small firms, and are more likely to develop a business application in-house. Firms are heterogeneous in their application needs and in their scale of operation, and software vendors can price discriminate based on firm's size. To the best of our knowledge, this is the first research to analytically model these aspects of the software industry.

We model a market with two types of firms: small firms that can not afford in-house development, and so buy the base-software only if they find a packaged application that provides a close match to their needs; and large firms that buy the base-software regardless of the number of ISVs (applications) in the market. We consider ISV entry costs and find the SZPE (symmetric zero profit equilibrium) number of ISVs, and under which conditions the base-software producer subsidizes ISVs. We show that when subsidies are optimal, the base-software producer subsidizes only the minimum number of ISVs required to serve all small firms. The base-software producer never finds it optimal to subsidize ISVs' entry in order to increase the number of ISVs in the market and eliminate in-house development by large firms. In addition, it is never optimal for the base-software producer to charge access fees from ISVs.

As mentioned in Gawer and Cusumano (2002): "what complements to make inside and what to leave to external firms is probably the single most important issue that platform leaders have to decide--and keep deciding." We show that if the number of ISVs in the market is such that the market is not covered (i.e., some small firms do not buy the base-software), integration of all ISVs, so that the base producer sells the base-software and the applications, increases user-firms' surplus and creates additional value for the base-software producer besides capturing the ISVs' profit. That is, the base producer's profit from selling the base-software and the applications is larger than the sum of the profits before the integration (base producer's profit from selling only base-software, and ISVs' profit from selling applications).

The base-software producer can choose to enter the applications market and prevent entry of ISVs. We find that as entry cost decreases, the market structure (which is dictated by the base producer's strategy) shifts from no packaged applications, to base-software producer's applications, to non-subsidized ISVs selling applications. This suggests that due to learning effects and reusability of software code, which decrease development cost, the market for independent software vendors selling specialized business applications should grow. As development cost decreases, the base-software producer would switch from selling applications to developing a network of ISVs, and the number of specialized applications offered in the market would increase.

This chapter is organized in the following manner: In § 5.2, I present the spatial model for the specialized applications market, find the optimal menu of prices when software companies can price discriminate based on firm's size, and determine the SZPE number of ISVs in the market. I then show under which conditions the base-software producer subsidizes entry and find the optimal subsidy and resulting number of ISVs. In § 5.3, I examine the case in which the base-software producer integrates all applications, and show when integration increases the total industry profit and user-firms' surplus. I conclude in § 5.4 and present most of the Proofs in § 5.5.

5.2 The Model

Most business applications operate in conjunction with base-software, such as an operating system (OS) or a relational database management system (RDBMS). Firms can choose whether to develop the business application in-house or buy a packaged application, but in either case they need to buy the base-software from the base-software producer. It can not be developed in-house.

Three factors affect the relative values of purchasing a packaged application and in-house development: fit of the software to the firm's business process, firm's operation scale, and the in-house development cost. A firm that chooses to develop a specialized application in-house incurs the development cost, which may be significant, but the resulting application would match the organization's needs and capture its unique business processes. A firm that decides to buy a packaged application might incur a misfit cost, because the packaged application usually differs from the firm's ideal application. The misfit cost, from using the packaged application, increases with the size (operation scale) of the firm, and so smaller firms which have smaller scale and lower frequency of decision-making incur a lower misfit cost.

The setting in this research involves a circular spatial market, introduced by Salop (1979). Firms' ideal applications are assumed to be located with uniform density on a circle, which without loss of generality is assumed to have a circumference of 2. ISVs (applications) are located at equal distances from each other on the circumference.

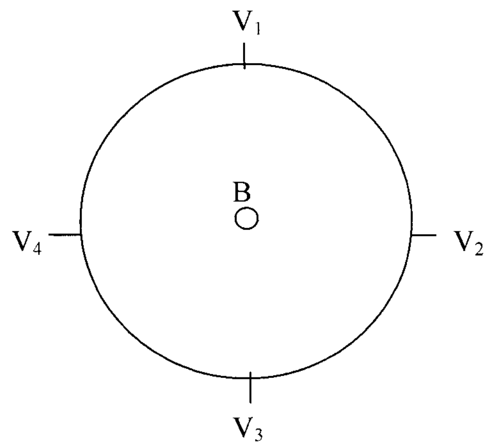

Figure 5.1: Spatial setting of the circular market

The base-software, *B*, has no market "location" in the applications–space. It is useful, however, to think of *B* as being located at the center of the circle, because in our model all firms would incur the same development cost if they develop a specialized application in-house, which implies that the base-software is not "closer" to some applications than to others. Notice that this setting also implies that ISVs in different locations have the same entry cost, the cost of developing a packaged application.

Figure 5.2 shows the different costs components for in-house development and for using a packaged application.

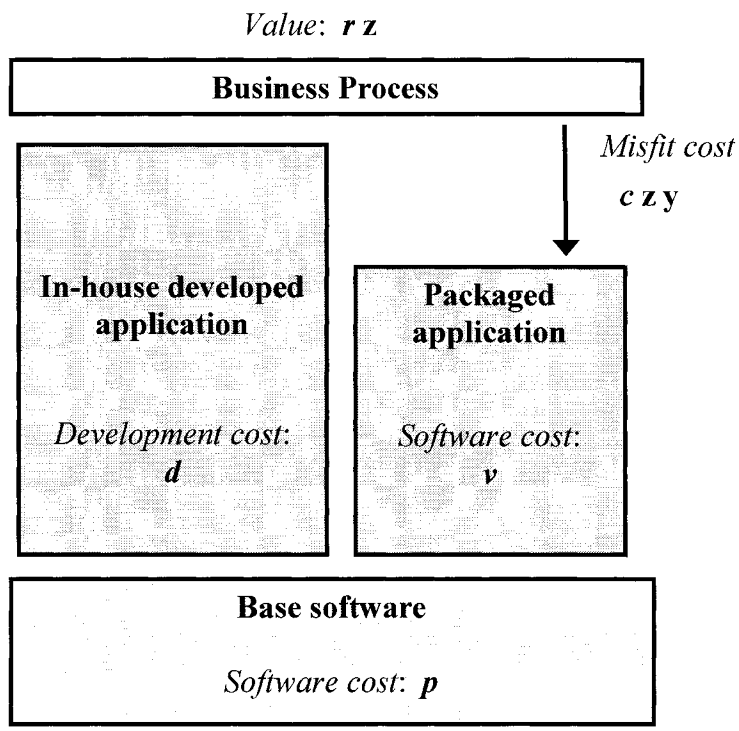

Figure 5.2: Cost of in-house development and cost of using a packaged application

As a basic requirement the firm needs to acquire the base-software, which can include an operating system (OS), a database management system (DBMS) etc. Firms buy base-software from a base-software producer with cost of $\$p$, which can be discriminatory based on firm's size. When developing the application in house (left side of Figure 5.2), the firm incurs the in-house development cost, $\$d$. The business application streamlines and automates the business process and so creates value, rz, for the firm which is increasing in the firm's scale of operation, z. Therefore, when a firm buys the base-software from the base producer and develops in-house the application required to achieve perfect fit with its needs, the net value to the firm is:

$$U(in-house) = rz - p - d\,. \tag{5.1}$$

If the firm chooses to buy a packaged application (right side in Figure 5.2), it purchases the base-software from the base-software producer for price $\$p$, and the application from the ISV for price $\$v$. The business application streamlines and automates the business process and so creates value to the firm, rz, minus a misfit cost, since the packaged application differs from the firm's ideal application. Hence, when a firm chooses to buy a packaged application from an ISV, its surplus is given by:

$$U(ISV) = z(r - cy) - p - v\,, \tag{5.2}$$

where y is the distance between the firm's ideal application and the packaged application, measured as the length of the arc between the two relevant locations, and cz is the marginal misfit cost which is linearly increasing with the firm's size (operation scale). Table 5.1 summarizes the notation used throughout the chapter.

r	value of business application per unit scale.
z	firm's scale of operation.
c	marginal misfit cost per unit scale.
d	application development cost (borne by a firm).
$p(z)$	price of base-software.
$v(z)$	price of ISV's application.
m	number of ISVs in the market.
αd	entry cost for ISV.
m^e	number of ISVs in the symmetric zero profit equilibrium (no subsidy).
n	number of applications owned by the base producer (in a market with no ISVs).

Table 5.1: Notation for Chapter 5

The business application streamlines and automates the business process, and therefore, the higher the transaction rate is, the higher is the value of the application to the user-firm. In this model we assume that the value of the business application is linearly increasing with the firm's operation scale. Nonlinearity, and specifically the assumption that the value of the business application is convex in the firm's operation scale, can be examined in future research. In addition, while the value of the business application increases with the operation scale (transaction volume) of the firm, we assume that the in-house development cost, incurred by the user-firm, is independent of the firm's operation scale. That is, the effort required to write the necessary software code does not depend on the future number of transactions. Notice that a model with a linear in-house development cost, $d+kz$, is equivalent to a model with a constant in-house development cost, d, but with a lower value per unit scale from applications developed in-house ($r-k$ value per unit scale from an in-house developed application versus r value per unit scale from a packaged application).

5.2.1. The Price Setting Game

We assume there are m, evenly spaced, ISVs in the circular market depicted in Figure 5.1. Entry costs are not relevant for the pricing game. The timing of the game captures the market power of the base-software producer: the base producer sets prices $p(z)$ first, and then ISVs set prices $v(z)$. Software companies can price discriminate based on firms' size.

Lemma 5.1: *In a circular market with m evenly spaced ISVs and a single base-software producer, who has first mover advantage in setting prices, the resulting prices and profits from firms of size z, are given by*

$$[p(z),\ v(z)] = \begin{cases} [0.5rz,\ 0.25rz] & \text{if } m \le 4c/r \text{ and } z < \dfrac{8cd}{8cr\text{-}mr^2} \\ [rz - 2cz/m,\ cz/m] & \text{if } m > 4c/r \text{ and } z < md/2c \\ [rz\text{-}d,\ 0.5d] & \text{else} \end{cases} \tag{5.3}$$

$$[\pi_B(z),\ \pi_v(z)] = \begin{cases} \left[\dfrac{mr^2z}{4c},\ \dfrac{r^2z}{8c}\right] & \text{if } m \le 4c/r \text{ and } z < \dfrac{8cd}{8cr\text{-}mr^2} \\ \left[2z(r - 2c/m),\ 2cz/m^2\right] & \text{if } m > 4c/r \text{ and } z < md/2c \\ \left[2(rz\text{-}d),\ d^2/2cz\right] & \text{else} \end{cases} \tag{5.4}$$

The proof is Section 5.5.

The above prices and profits are continuous in m for a given z. In Figure 5.3 we depict the prices, given in Lemma 5.1, and the resulting market structure in the (m, z) space. Notice that ISVs can capture at most half of the in-house development cost, that is $v(z)$ is never larger than $d/2$. In other words, the price that a firm has to pay for a packaged application is always less than half the cost of building the application in-house.

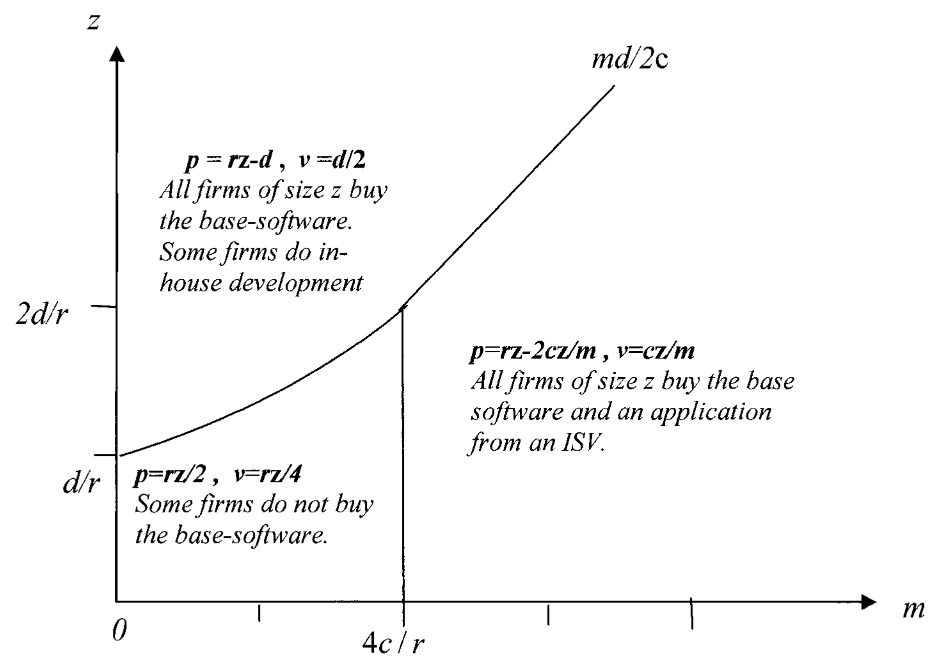

Figure 5.3: Competition and prices as functions of the number of ISVs, *m*, and firms' size, *z*, for given values of *c*, *d* and *r*.

We can now measure channel power, defined as the proportion of channel profits that accrue to each of the channel members (Kadiyali et al, 2000). The proportion of industry profit reaped by the base-software producer, from firms of size *z*, is given by:

$$\frac{\pi_B(z)}{\pi_B(z)+m\pi_v(z)}=\begin{cases}\frac{2}{3} & \text{if } m\le 4c/r \text{ and } z<\frac{8cd}{8cr\text{-}mr^2}\\ 1-\frac{c}{mr-c} & \text{if } m>4c/r \text{ and } z<\frac{md}{2c}\\ \frac{4cz(rz-d)}{4cz(rz-d)+md^2} & \text{else}\end{cases}.$$

As the number of ISVs increases, the total industry profit increases (see Equation 5.4). The base producer's channel power is decreasing with the number of ISVs, *m*, when ISVs compete with in-house development by firms, and is increasing with the number of ISVs when ISVs compete with each other. Size of user-firms affects channel power only when firms choose between a packaged application and in-house development, in which case an increase in firms' size, z, increases the base producer's channel power but decreases each ISV's channel power. The base producer has at least

2/3 of the industry's profit from small firms (firms that can not consider in-house development), and he would like to increase the number of applications sold to small firms, since then not only does total industry profit increase, but his fraction of that pie increases.

5.2.2. The Entry Equilibrium

For the rest of the chapter we assume that there are only two values of z, denoted as z_L and z_H. Specifically, we assume that firms can be either large, with operation scale $z_H > 2d/r$, or small, with operation scale $z_L < d/r$. Hence, regardless of the number of ISVs, *m*, large firms at all locations buy the base-software, and small firms never do in-house development of business applications. If large firms and small firms are uniformly distributed over the circle circumference, each with density 2, and entry cost for an ISV (the cost of developing the packaged application) is αd, then based on Lemma 5.1, an ISV's total profit from both types of firms is given by:

$$\Pi_v(m) = \begin{cases} \dfrac{r^2 z_L}{8c} + \dfrac{d^2}{2cz_H} - \alpha d & \text{if } m \le 4c/r \\ \dfrac{2cz_L}{m^2} + \dfrac{d^2}{2cz_H} - \alpha d & \text{if } 4c/r < m \le 2cz_H/d \\ \dfrac{2c(z_L + z_H)}{m^2} - \alpha d & \text{if } m > 2cz_H/d \end{cases} . \tag{5.5}$$

Lemma 5.2: *When the ISVs have entry cost αd, the symmetric zero profit equilibrium number of ISVs is given by*

$$m^e = \begin{cases} 0 & \text{if } \alpha d > \dfrac{r^2 z_L}{8c} + \dfrac{d^2}{2cz_H} \\ \sqrt{\dfrac{4c^2 z_L z_H}{d(2cz_H\alpha - d)}} & \text{if } \dfrac{d^2(z_L + z_H)}{2cz_H^2} < \alpha d \le \dfrac{r^2 z_L}{8c} + \dfrac{d^2}{2cz_H} \\ \sqrt{\dfrac{2c(z_L + z_H)}{\alpha d}} & \text{if } \alpha d \le \dfrac{d^2(z_L + z_H)}{2cz_H^2} \end{cases} .[8] \tag{5.6}$$

Proof is in Section 5.5.

[8] Notice that $2c\alpha z_H > d$ whenever $\alpha > d(z_L + z_H)/(2cz_H^2)$

Notice that if $\alpha d > \frac{r^2 z_L}{8c} + \frac{d^2}{2cz_H}$ then there are no ISVs and small firms never buy the base-software, while if $\alpha d < \frac{r^2 z_L}{8c} + \frac{d^2}{2cz_H}$, then in equilibrium there are at least $4c/r$ ISVs, enough to serve all small firms in all locations. We never have an equilibrium in which the market of small firms is only partially covered (only some small firms have an application close enough to their ideal application).

Next we show that if ISVs can not enter the market (high entry cost), the base-software producer might find it optimal to subsidize ISVs' entry, sharing his gains from selling the base-software to small firms. In addition, the base producer never finds it optimal to charge access fees from ISVs.

5.2.3. Subsidizing ISVs entry (Subsidy or Fee)

The base producer can control the number of applications in the market by subsidizing ISVs' entry or charging fees for access to information on the base-software's interfaces. A subsidy indicates that the base producer treats ISVs as partners, while fees suggest channel conflict and competition. The base producer chooses the number of ISVs that maximizes his profit from selling the base-software minus the subsidy he needs to pay (or plus the fees charged), which, based on Lemma 5.1, is given by:

$$\Pi_B(m) = \begin{cases} \frac{r^2 z_L m}{4c} + 2(rz_H - d) - \gamma[m]\alpha dm & \text{if } m \le 4c/r \\ 2z_L(r - 2c/m) + 2(rz_H - d) - \gamma[m]\alpha dm & \text{if } 4c/r < m \le 2cz_H/d \\ 2(r\text{-}2c/m)(z_L + z_H) - \gamma[m]\alpha dm & \text{if } m > 2cz_H/d \end{cases} \quad (5.7)$$

The base producer's profit is continuous in m, and $\gamma[m]$ is the subsidy as fraction of application development cost (when positive) or fee (when negative) required to have a market with m ISVs. $\gamma[m]$ is given by

$$\gamma[m] = -\frac{\Pi_v(m)}{\alpha d}, \quad (5.8)$$

where $\Pi_v(m)$ is given in Equation 5.5. Notice that $\gamma[m] \le 1$ for all values of m. Clearly when $\Pi_v(m) > 0$, the base producer needs to charge fees in order to limit the number of

ISVs in the market to m, while when $\Pi_v(m)<0$, the base producer needs to subsidize ISVs' entry (pay a fraction of the entry cost) in order to encourage m ISVs to enter the market. Substituting for $\gamma[m]$ in Equation 5.7 and taking the derivative with respect to m we have:

$$\frac{\partial \Pi_B(m)}{\partial m} = \begin{cases} \Pi_v(m) + \dfrac{2r^2 z_L}{8c} & \text{if } m < 4c/r \\ \Pi_v(m) & \text{if } m > 4c/r \end{cases} \tag{5.9}$$

and $\partial^2 \Pi_B(m)/\partial m^2 = \partial \Pi_v(m)/\partial m \leq 0$ (see proof of Lemma 5.2), so that the base producer's profit is concave in m. If the SZPE number of ISVs (with no subsidy or fee) is such that $m^e > 4c/r$, then m^e also maximizes the base producer's profit, that is $\partial \Pi_B(m^e)/\partial m = 0$, and there is no subsidy or fee. While if $m^e \leq 4c/r$ (which, according to Lemma 5.2, happens only if $m^e=0$), the base producer subsidizes $4c/r$ ISVs if and only if $\left.\frac{\partial \Pi_B(m)}{\partial m}\right|_{m=4c/r} = \Pi_v(4c/r) + \frac{2r^2 z_L}{8c} > 0$, because $\Pi_v(m)$ is not a function of m for $m \leq 4c/r$.

Proposition 5.1: *The base producer subsidizes entry if an only if*

1. *With no subsidy there are no ISVs in the market, i.e.* $\frac{r^2 z_L}{8c} + \frac{d^2}{2cz_H} < \alpha d$, *and*
2. $\frac{3r^2 z_L}{8c} + \frac{d^2}{2cz_H} > \alpha d$,

in which case the base-software producer subsidizes the entry of 4c/r ISVs.

The base producer never charges access fees from the ISVs. Figure 5.4 depicts the number of ISVs in the market, when the base-software producer can subsidize entry, as function of z_L and α.

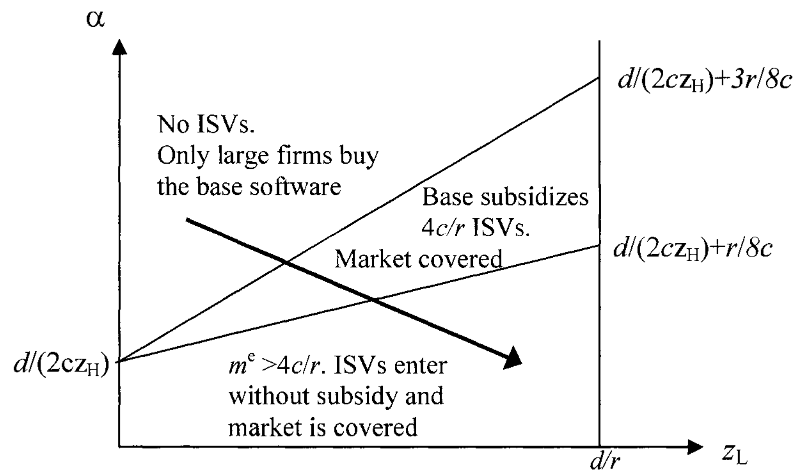

Figure 5.4: Number of ISVs in the market when base producer can subsidize entry.

Our analysis suggests how we can anticipate these software markets to evolve. As development costs decrease due to learning affects, reusability of software code and experience, markets with no ISVs (no packaged applications) transform into markets with ISVs which are subsidized by the base-software producer.

5.3 Integrated Applications

In this section we first find the optimal prices, $p(z)$ and $v(z)$, when the base-software producer sells the base software and n applications (no ISVs in the market). The n applications are located at equal distances from each other on the circumference of the circular market. We then examine whether the base-software producer should integrate ISVs, and what is the optimal number of base-owned applications, n, as a function of the development cost. As described in Proposition 5.1, if $m^e = 0$, the base producer might subsidize $4c/r$ ISVs. In this section, we examine whether the base producer prefers to develop and sell its own packaged applications rather than subsidize ISVs.

We start by solving for the optimal prices charged by the base producer, for a given number of applications, n. Applications development costs are not relevant for the pricing decision. Since the base-software producer sells the base-software and the business applications, he prices them simultaneously.

Lemma 5.3: *When the base-software producer sells n applications, the total price of base-software and application, T, charged from firms with z < d/r, and the profit from such firms are given by:*

$$T(z)=\begin{cases} rz/2 & \text{if } n<2c/r \\ rz-cz/n & \text{else} \end{cases} \quad \text{and} \quad \pi(z)=\begin{cases} r^2zn/(2c) & \text{if } n<2c/r \\ 2z(r-c/n) & \text{else} \end{cases} \text{, respectively.}$$

The prices of base-software and application offered to firms with z >d/r, are given by p(z)= rz-d and v(z) =d/2 when z > dn/(2c) and by T(z) = rz-cz/n otherwise.

Figure 5.5 presents the results of Lemma 5.3 graphically.

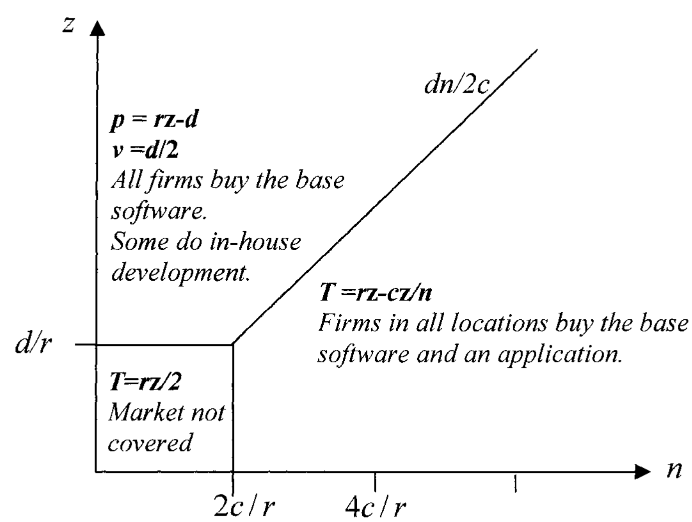

Figure 5.5: Market coverage and prices as functions of the number of integrated applications, *n*, and end-user firms' size, *z*, for given values of *c*, *d* and *r*.

Comparing Figure 5.3 and Figure 5.5 we see that if (and only if) the number of ISVs, *m*, is such that $m < 4c/r$, value is created when the base producer integrates the *m* applications (ignoring the cost of the integration). That is, the base-software producer's profit from selling the base-software and the *m* applications is larger than the sum of the base producer's profit from selling only base-software and the profits of *m* ISVs.

Proposition 5.2: *Integration of m applications increases user-firms' surplus and the total profit from selling software (base-software and applications) if and only if the number of ISVs in the market is such that m <4c/r, that is iff market is not covered.*

Proof of Proposition 5.2: Clearly, the surplus of large firms, $z_H > 2d/r$, is the same whether the *m* applications are offered by the ISVs or owned by the base-software producer (since prices and coverage under the two cases are the same). For small firms, $z_L < d/r$, the above holds if and only if $m > 4c/r$, since then small firms at all locations buy the base-software and an application, with total cost of $rz-cz/m$, whether the applications are sold by the base producer or by the ISVs. When $2c/r < m < 4c/r$, if the m applications are integrated, all small firms buy the base-software and an application, with total cost of $rz-cz/m$, while if applications are sold by ISVs, the total price of the software ($3rz/4$) is higher and only some of the small firms can buy the software. Hence, firms' surplus is higher when applications are integrated. When $m < 2c/r$, the price of the system (base + application) is lower when the applications are sold by the base producer and more small firms can buy the software. Hence, again firms' surplus is higher when applications are integrated.

To show that value is created by integration iff $m < 4c/r$, we use Equation 5.4 and Lemma 5.3. If $n < 2c/r$, a base producer that owns n applications makes $r^2 z_L n/(2c)$ from selling the base-software and applications. If the n applications are sold by n ISVs, the base producer makes $r^2 z_L n/(4c)$ and each ISV makes $r^2 z_L/(8c)$. Value is created by integration because $r^2 z_L n/(2c) > 3r^2 z_L n/(8c)$. If $2c/r < n < 4c/r$, a base producer that owns n applications makes $2z_L(r - c/m)$, and $2z_L(r - c/m) > 3r^2 n z_L/(8c)$ if and only if $3r^2n^2 - 16rnc + 16c^2 < 0$, but the LHS of the last inequality equals $(rn - 4c)(3rn - 4c)$ which is negative because $2c/r < n < 4c/r$. Therefore, value is created by integration (besides capturing the ISVs' profit). Similarly, it is easy to show that when $m > 4c/r$, by integrating the m applications the base-software producer only captures the ISVs' profit (no value is created), and this holds for any value of m, regarding ISVs profit from large firms. ■

According to Proposition 5.2, if the number of ISVs in the market already exceeded $4c/r$ there is less incentive for integration. Firms' surplus would not increase, and the base producer would only capture the ISVs' profit, and so would not have "extra value" in order to purchase the ISVs at their worth.

Comparing Figure 5.3 and Figure 5.5 we also learn that when applications are integrated, fewer applications are needed for all small firms ($z_L < d/r$), at all locations, to buy the base-software. Specifically, the entire market of small firms can be served with half the number of applications, as is summarized in Proposition 5.3.

Proposition 5.3: *When applications are integrated (sold by the base-software producer), the minimum number of applications required to serve small firms at all locations is 2c/r, while when applications are sold by ISVs, 4c/r applications are required to serve all small firms.*

The base-software producer's profit increases with the number of applications, *n*, that he sells. However, we must consider the cost the base producer incurs when developing or purchasing the applications. The cost of obtaining *n* applications, *C*(n), is clearly increasing in *n*, and might be concave in *n* due to learning affects or reusability of software code. However, to make the comparison with the case of applications sold by ISVs more compelling, we assume that $C(n)=\alpha dn$, so that the cost of having *n* applications is linearly increasing with the number of applications. Based on Lemma 5.3, the base producer's profit from selling the base-software and *n* applications is

$$\Pi_B(n) = \begin{cases} \dfrac{nr^2 z_L}{2c} + 2(rz_H - d) + \dfrac{nd^2}{2cz_H} - C(n) & \text{if } n \le 2c/r \\ 2z_L\left(r - \dfrac{c}{n}\right) + 2(rz_H - d) + \dfrac{nd^2}{2cz_H} - C(n) & \text{if } 2c/r < n \le 2cz_H/d \\ 2z_L(r - c/n) + 2z_H(r - c/n) - C(n) & \text{if } n > 2cz_H/d \end{cases} \quad (5.10)$$

Lemma 5.4: *The number of integrated applications that maximizes the base producer's profit is given by:*

$$n_{opt} = \begin{cases} 0 & \text{if } \alpha d > \dfrac{r^2 z_L}{2c} + \dfrac{d^2}{2cz_H} \\ \sqrt{\dfrac{4c^2 z_L z_H}{d(2cz_H\alpha - d)}} & \text{if } \dfrac{d^2(z_L + z_H)}{2cz_H^2} < \alpha d < \dfrac{r^2 z_L}{2c} + \dfrac{d^2}{2cz_H}. \\ \sqrt{\dfrac{2c(z_L + z_H)}{\alpha d}} & \text{if } \alpha d < \dfrac{d^2(z_L + z_H)}{2cz_H^2} \end{cases} \quad (5.11)$$

Assuming the base producer can prevent ISVs from entering the market, we next find the resulting market structure (whether there are n_{opt} applications owned by the base producer, m^e ISVs, or 4*c*/*r* subsidized ISVs) as a function of the development cost, αd.

We assume that if the base producer's profit is the same whether he owns n_{opt} applications or has ISVs, he prefers to have ISVs.

Proposition 5.4: *The number of applications in the market, and the ownership of the applications, when either all applications are owned by the base producer or all applications are sold by ISVs, is the following:*

$m^e > 4c/r$ *ISVs*	*if* $\alpha d < r^2 z_L/(8c) + d^2/(2cz_H)$
n_{opt} *base owned applications*	*if* $r^2 z_L/(8c) + d^2/(2cz_H) < \alpha d < r^2 z_L/(2c) + d^2/(2cz_H)$
no applications in market	*else*

n_{opt} *and* m^e*, are given by Equation 5.11 and Equation 5.6 respectively.*

Proof: First notice that when ISVs can enter the market with no subsidy, the optimal number of owned applications, n_{opt}, is the same as the number of ISVs in the SZPE, m^e, and the profit of the base-software producer is the same whether he chooses to have only owned applications or only ISVs. Hence, when $\alpha d < r^2 z_L/8c + d^2/(2cz_H)$, there will be $m^e > 0$ ISVs in the market. On the other extreme, when $\alpha d > r^2 z_L/2c + d^2/(2cz_H)$ there are no applications in the market, since the base producer does not subsidize ISVs nor develop applications. When $3r^2 z_L/8c + d^2/(2cz_H) < \alpha d < 4r^2 z_L/8c + d^2/(2cz_H)$, the base-producer develops and sells n_{opt} packaged application, and does not subsidize ISVs entry (See Proposition 5.1 and Lemma 5.4).

When $r^2 z_L/8c + d^2/(2cz_H) < \alpha d < 3r^2 z_L/8c + d^2/(2cz_H)$, we need to compare the base producer's profit when selling n_{opt} applications with his profit from subsidizing $4c/r$ ISVs. Since $\alpha d > r^2 z_L/8c + d^2/(2cz_H)$, the optimal number of owned applications satisfies $\frac{2c}{r} < \sqrt{\frac{4c^2 z_L z_H}{d(2cz_H\alpha - d)}} < \frac{4c}{r}$, so the number of owned applications is smaller than the number of subsidized applications. The base producer's profit from selling 4c/r applications or from having 4c/r ISVs is the same. Since the number of owned applications is smaller than $4c/r$, the base producer's profit from owning applications is higher then his profit from subsidizing ISVs. ■

When considering the two extreme cases – only ISVs sell applications or only the base producer sells applications (they do not compete in the applications market), we find that if the base producer has the tools to develop applications, then he does not subsidize ISVs, but instead develops and sells applications itself, even when there are no "economies of scale" (that is even when there are no learning effects and reusability of software code). However, the base producer will not prevent ISVs entry when ISVs can be profitable without a subsidy. Firms would be better off if the base producer subsidized ISVs entry since then the variety of applications in the market would be higher (lower misfit costs).

5.6 Conclusions

In this Chapter we model the interactions between a base-software producer, ISVs selling specialized business applications, and firms with heterogeneous application needs and heterogeneous operation scale. We model the tradeoffs firms face when choosing between in-house development of business applications and purchasing packaged applications, and solve for the optimal prices charged by the base-software producer and ISVs in the vertical pricing game. We show that small firms buy packaged applications, while large firms are more likely to choose in-house development due to potential high misfit cost when using packaged applications.

We consider ISVs entry cost and find the symmetric zero profit equilibrium (SZPE) number of ISVs. We show that in equilibrium either all small firms can buy the base-software and an application, or there are no ISVs in the market. We find the conditions under which the base producer subsidizes ISVs entry and prove that he never charges access fees from ISVs.

Considering integration of ISVs by the base producer, we show that if some small firms can not find an application close enough to their needs and so do not buy the base-software, the base producer can increase firms' surplus and create value (increase total industry profit) by integrating ISVs. That is, as long as market is not covered (some firms do not buy the base-software) value is created by integration.

We find that if the base-producer can develop and sell applications, he would never subsidize ISVs entry. However, the base producer does not enter the applications market when ISVs can be profitable without a subsidy. We show that as development cost decreases the base-producer prefers having ISVs to selling applications itself, which explains the growing number of ISVs in the software industry (see Oracle website for Oracle's ISVs network).

5.5 Proofs

Proof of Lemma 5.1

The base-software producer sets price p(z) first, and then ISVs set price v(z). Hence, for each possible p(z) we find the ISVs' best reaction, $v(z)$, to determine the base producer's profit as a function of p(z).

ISVs compete with each other if and only if $0 \le v \le \text{Min}\ [rz\text{-}p\text{-}cz/m,\ d\text{-}cz/m]$, so that a firm at distance $1/m$ from closest ISV has positive surplus from using the ISV and this surplus is larger than the firm's surplus from in-house development. Hence, $z < md/c$ is necessary for ISVs to have the option of spatial competition. Denoting M=Min $[rz\text{-}p\text{-}cz/m,\ d\text{-}cz/m]$, and assuming symmetric ISVs, an ISV's profit is given by

$$\pi_v = \begin{cases} v2/m & \text{if } 0 \le v \le M \\ v2\left(\dfrac{rz-p-v}{cz}\right) & \text{if } rz\text{-}p\text{-}cz/m < v < rz\text{-}p \text{ and } p > rz\text{-}d \\ v2(d\text{-}v)/cz & \text{if } M < v < d \text{ and } p \le rz\text{-}d \\ 0 & \text{else} \end{cases} \tag{5.12}$$

When z $>md/c$, so that $M<0$, the ISV's profit is maximized at $0.5(rz\text{-}p)$ if $p>rz\text{-}d$ and at $d/2$ otherwise. When z $< md/$c, consider first $p>rz\text{-}d$ (so that $M=rz\text{-}p\text{-}cz/m$). Then, the ISV's profit is continuous in v, linearly increasing in v for $v \le$ M, and strictly concave in v otherwise, and is increasing at $v=M$ iff $0.5(rz\text{-}p)> rz\text{-}p\text{-}cz/m$, that is iff $p>rz\text{-}2cz/m$. If $p \le rz\text{-}d$, the ISV's profit is continuous at $v=M=d\text{-}cz/m$ and is increasing at $v=d\text{-}cz/m$ iff $d/2>d\text{-}cz/m$, that is iff $z>md/2c$. Hence, ISV's best response is given by:

$$v(z) = \begin{cases} 0.5(rz-p). & \text{if } z > md/c \text{ and } p > rz\text{-}d \\ d/2 & \text{if } z > md/c \text{ and } p \le rz\text{-}d \\ 0.5(rz\text{-}p) & \text{if } z < md/c \text{ and } p > rz\text{-}d \text{ and } p > rz\text{-}2cz/m \\ rz - p\text{-}cz/m & \text{if } z < md/c \text{ and } p > rz\text{-}d \text{ and } p < rz\text{-}2cz/m \\ d/2 & \text{if } md/2c < z < md/c \text{ and } p \le rz\text{-}d \\ d - \text{cz/m} & \text{if } z < md/2c \text{ and } p \le rz\text{-}d \end{cases}, \tag{5.13}$$

and the Base producer's profit is given by:

$$\pi_B = \begin{cases} p2m\left(\dfrac{rz-p}{2cz}\right) & \text{if } [p > rz\text{-}d] \text{ and } [(z > md/c) \text{ or } (z < md/c \text{ and } p > rz\text{-}2cz/m)] \\ 2p & \text{else} \end{cases} \quad (5.14)$$

When z >*md*/2*c*, *rz-d >rz-2cz/m*, and so: $\pi_B = 2pm(rz-p)/(2cz)$ if $p>rz-d$ and $\pi_B=2p$ otherwise. The base producer's profit is not continuous at *p*=*rz*-d. When z>*d*/*r*, we have

$\pi_B(rz-d+\varepsilon) \xrightarrow{\varepsilon \to 0^+} 2m(rz-d)d/(2cz)$ and so $\pi_B(rz-d+\varepsilon) < \pi_B(rz-d) = 2(rz-d)$ for every $\varepsilon \to 0^+$ if and only if z > *md*/2c and the value of the profit function to the right of *rz-d* is always smaller than its value at *rz*-d. Figures 5.6, 5.7 and 5.8 describe the base-producer's profit as a function of his price, taking into account the ISV's best response, when *z*<d/*r*, *d/r<z<2d/r* and z >*2d/r*, respectively. If z <*d*/r then the optimal base-software price is given by *p*(z)=0.5*rz*, as exhibited in Figure 5.6. If *z* >2*d*/r then the optimal base-software price is given by *p*(*z*)=*rz*-d, as is exhibited in Figure 5.8. If d/*r* <z<2*d*/r, we need to compare the base-software producer's profit when *p*(*z*)=0.5*rz* with his profit when *p*(z)=*rz-d*; the first is larger iff $z < \dfrac{8cd}{r(8c-rm)}$.

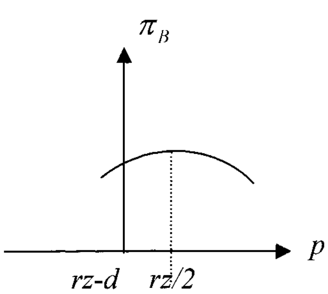

Figure 5.6: *z<d/r*

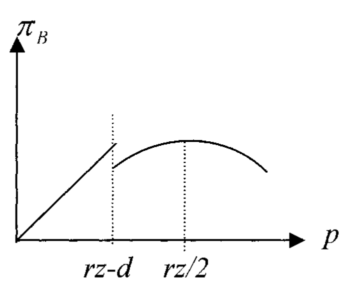

Figure 5.7: *d/r <z<2d/r*

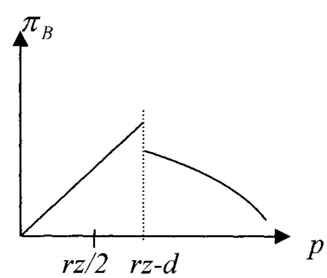

Figure 5.8: *z>2d/r*

Notice, that for *z* > *md*/2*c* we have $\dfrac{d}{r} < \dfrac{8cd}{r(8c-rm)} < \dfrac{2d}{r}$, and so when z > *md/2c*, for $z < 8cd/(8cr - r^2m)$ the optimal *p*(z) is *rz*/2 , ISVs have spatial monopoly, and *v*(*z*) is *rz*/4. For $z > 8cd/(8cr - r^2m)$ the optimal *p*(z) is *rz-d*, ISVs compete with the "direct sales" (in-house development by firms) and *v*(z) is *d*/2.

When z <*md/2c* using Equation 5.14 the base producer's profit is given by *2p* when $p \le$ *rz-2cz/m* and by $\pi_B=2pm(rz-p)/(2cz)$ otherwise, and is continuous in *p*. The base producer's profit is linearly increasing in *p* for $p \le$ *rz-2cz/m* and strictly concave in p otherwise. Thus p(z)=rz/2 iff π_B is increasing at $p = rz$-2cz/m that is iff $rz/2 > rz-2cz/m$ ($m < 4c/r$), and p(z)=rz-2cz/m otherwise.

We summarize that the optimal p(z), and the resulting v(z) are given by:

$$[p(z),\ v(z)] = \begin{cases} [0.5rz,\ 0.25rz] & \text{if } m \le 4c/r \text{ and } z < \dfrac{8cd}{8cr - mr^2} \\ [rz - 2cz/m,\ cz/m] & \text{if } m > 4c/r \text{ and } z < md/2c \\ [rz - d,\ 0.5d] & \text{else} \end{cases}.$$

In the first range of parameters values $p(z) > rz-d$ and ISVs have spatial monopoly. In the second range of parameters values, $p > rz-d$ and ISVs compete with each other; and in the last range of parameters values, $p = rz-d$ and ISVs compete with in-house development by firms. Substituting the optimal prices in the relevant expressions from Equations 5.12 and 5.14 we find the resulting profits:

$$[\pi_B(z),\ \pi_v(z)] = \begin{cases} \left[mr^2 z/4c,\ r^2 z/8c \right] & \text{if } m \le 4c/r \text{ and } z < \dfrac{8cd}{8cr - mr^2} \\ \left[2z(r - 2c/m),\ 2cz/m^2 \right] & \text{if } m > 4c/r \text{ and } z < md/2c \\ \left[2(rz-d),\ d^2/2cz \right] & \text{else} \end{cases}. \blacksquare$$

Proof of Lemma 5.2

According to Equation 5.5, for $m \le 4c/r$ the ISV's profit is constant in m. For $4c/r < m \le 2cz_H/d$, the ISV's profit is strictly decreasing and convex in m as $\partial \Pi_v / \partial m = -4cz_L/m^3$ and $\partial^2 \Pi_v / \partial m^2 = 12cz_L/m^4$. For $m > 2cz_H/d$ the ISV's profit is strictly decreasing and convex in m as $\partial \Pi_v / \partial m = -4c(z_L + z_H)/m^3$ and $\partial^2 \Pi_v / \partial m^2 = 12c(z_L + z_H)/m^4$. In addition, it is easily shown that the ISV's profit is continuous in m.

If $\Pi_v(2cz_H/d) > 0$, that is if $\dfrac{d^2(z_L + z_H)}{2cz_H^2} - \alpha d > 0$, then $\dfrac{2c(z_L + z_H)}{m_e^2} = \alpha d$ and in the symmetric zero profit equilibrium (SZPE) firms at all locations, small and large, use the ISVs. If $\dfrac{d^2(z_L + z_H)}{2cz_H^2} - \alpha d < 0$ but $\Pi_v(4c/r) > 0$, that is $\dfrac{r^2 z_L}{8c} + d\left(\dfrac{d}{2cz_H} - \alpha\right) > 0$, then

$\frac{2cz_L}{m_e^2}+d\left(\frac{d}{2cz_H}-\alpha\right)=0$, and in the SZPE small firms at all locations use ISVs, and some large firms do in-house development. If $\frac{r^2z_L}{8c}+\text{d}\left(\frac{\text{d}}{2\text{cz}_\text{H}}-\alpha\right)$<0 then ISVs do not enter the market, $p(z_H) = rz_H - d$, and all large firms do in-house development. There is no profit for the base -software producer from small firms.■

Proof of Lemma 5.3

We use T to denote $(p+v)$. Here, we find the optimal prices, $p(z)$ and $v(z)$, offered by a base-software producer selling the base-software and n symmetric applications. The base producer prices base-software and applications simultaneously to maximize his total profit. If $p>rz-d$, then the base producer in fact bundles the base-software with an application at price $T>rz$-d.

If $rz-d<0$:

$$\pi_B(T)=\begin{cases} T2n\dfrac{rz-T}{cz} & \text{if } T>rz\text{-}cz/n \\ 2T & T\leq rz\text{-}cz/n \end{cases}$$

The profit is continuous in T, linearly increasing in T for $T \leq rz\text{-}cz/n$ and strictly concave in T otherwise. Hence, the optimal price is $T=Max\ \{rz/2,\ rz\text{-}cz/n\}$, that is, $T=rz/2$ and market is not covered if $n<2c/r$, and $T=rz\text{-}cz/n$ and market is covered otherwise.

If $rz-d>0$ and $rz-d>rz-cz/n$ (that is $z>d/r$ and $z>dn/c$):

$$\pi_B(p,v)=\begin{cases} (p+v)2n\dfrac{rz-p-v}{cz} & \text{if } p>rz-d \text{ and } T<rz \\ 2p+\text{v}2n\dfrac{d\text{-}v}{cz} & \text{if } p\leq rz\text{ - }d \text{ and } v<d \\ 2p & \text{if } p\leq rz\text{ - }d \text{ and } v>d \\ 0 & \text{else} \end{cases}$$

Notice that in this range of parameters values we can't have positive prices at which all firms use the ISVs, since that requires $v<d\text{-}cz/n$, but $d<\text{cz}/n$. Ignoring the constrain on p, the first expression is maximized at $T=rz/2$, with maximum value of $r^2zn/(2c)$. The second expression is maximized at $p=rz$-d and $v=d/2$, with value of $2(rz\text{-}d)+d^2n/(2cz)$.

$2(rz\text{-}d)+d^2n/(2cz) > r^2zn/(2c)$ if and only if $z <$ Min$\{d/r, dn/(4c\text{-}nr)\}$ or $z >$ Max$\{d/r, dn/(4c\text{-}nr)\}$. If $n < 2c/r$, then $d/r > dn/(4c\text{-}nr)$ and $2(rz\text{-}d)+d^2n/(2cz) > r^2zn/(2c)$ for all $z > d/r$, and the optimal prices are $p=rz\text{-}d$ and $v=d/2$. If $n > 2c/r$ then $z > dn/c > 2d/r$ and so $rz/2 < rz\text{-}d$ and $T2n(rz-T)/(cz)$ is decreasing in T for $T > rz\text{-}d$. $T2n(rz-T)/(cz)$ evaluated at $T=rz\text{-}d$ gives $(rz\text{-}d)2nd/(cz)$, which is smaller than $2(rz\text{-}d)+d^2n/(2cz)$, since $2dn/(cz) < 2$. Therefore $p = rz\text{-}d$ and $v=d/2$.

If $rz\text{-}cz/n > rz\text{-}d > 0$ (that is $z > d/r$ and $z < dn/c$) : The base producer's profit is given by

$$\pi_B(p,v) = \begin{cases} T2n\dfrac{rz-T}{cz} & \text{if } p > rz\text{-}cz/n \\ T2n\dfrac{rz-T}{cz} & \text{if } rz\text{-}d < p \le rz\text{-}cz/n \text{ and } T > rz\text{-}cz/n \\ 2T & \text{if } rz\text{-}d < p \le rz\text{-}cz/n \text{ and } T \le rz\text{-}cz/n \\ 2p + v2n\dfrac{d-v}{cz} & \text{if } p \le rz-d \text{ and } v > d\text{-}cz/n \\ 2T & \text{if } p \le rz-d \text{ and } v \le d\text{-}cz/n \end{cases}$$

In the range where profit is given by $2p+v2n(d\text{-}v)/(cz)$, the profit is maximized at $p=rz$-d and $v=d/2$, with value $2(rz\text{-}d)+d^2n/(2cz)$, if $d/2 > d\text{-}cz/n$ (that is if $z > nd/2c$), and at $p=rz\text{-}d$ and $v=d\text{-}cz/n$ otherwise (with value smaller than $2(rz\text{-}d)+d^2n/(2cz)$).

In the range where profit is given by $2T$, the profit is maximized at $T=z(r\text{-}c/n)$, with profit of $2z(r\text{-}c/n)$.

In the range where profit is given by $2p+v2n(d\text{-}v)/(cz)$, the profit is maximized at $p=rz$-d and $v=d/2$, with value $2(rz\text{-}d)+d^2n/(2cz)$, if $d/2 > d\text{-}cz/n$ (that is if $z > nd/2c$), and at $p=rz\text{-}d$ and $v=d\text{-}cz/n$ otherwise (in which case the profit is smaller than $2(rz\text{-}d)+d^2n/(2cz)$).

In the range where profit is given by $2T$, the profit is maximized at $T=z(r\text{-}c/n)$, with profit of $2z(r\text{-}c/n)$.

- **When $z < nd/2c$**: Since $z > d/r$ we have $n > 2c/r$ and so $rz/2 < rz\text{-}cz/n$. Hence, $T2n\dfrac{rz-T}{cz}$ is decreasing in T for $T > rz\text{-}c/n$. Also, $2p+v2n(d\text{-}v)/cz$ is decreasing in v for $v > d\text{-}cz/n$ and so never exceeds $2T$. Notice that $T2n\dfrac{rz-T}{cz} = 2T$ at $T=rz\text{-}cz/n$ and $2T$

$= 2p+v2n(d\text{-}v)/cz$ when $v=d\text{-}cz/n$ and $p=rz\text{-}d$. Hence, when $z < nd/2c$ the optimal price is $T=(rz\text{-}cz/n)$ and profit is given by $2(rz\text{-}cz/n)$.

- **When z > *nd*/2c**: $2(rz\text{-}d)+d^2n/(2cz) - 2(rz\text{-}cz/n) > 0$ because $4cz(rz\text{-}d)+ d^2n\text{-}4cz(rz\text{-}cz/n)= (2cz\text{-}nd)^2/n > 0$. Hence, pricing $p=(rz\text{-}d)$ and $v=d/2$, with some firms doing in-house development, yields a higher profit than pricing $T=rz\text{-}cz/n$ with all firms using the ISVs.
  - If $n < 2c/r$ then $rz/2 > rz\text{-}cz/n$ and $d/r > dn/(4c\text{-}nr)$. $T2n\frac{rz-T}{cz}$ is maximized at $rz/2$ and, as before, $\frac{r^2zn}{2c} > 2(rz-d)+n\frac{d^2}{2cz}$ if and only if $z \in \left[\frac{dn}{4c-nr}, \frac{d}{r}\right]$. But $z > d/r$ in this range of parameters and so $r^2zn/(2c) < 2(rz-d)+nd^2/(2cz)$. The optimal prices are $p=rz\text{-}d$ and $v=d/2$.
  - In $n > 2c/r$ then $rz/2 < rz\text{-}cz/n$ and so $T2n\frac{rz-T}{cz}$ is decreasing in T for $T > rz\text{-}cz/n$, and never exceeds the value $2(rz\text{-}cz/n)$. In addition, we already showed that $2(rz-d)+nd^2/(2cz) > 2(rz-cz/n)$ when $z > nd/2c$. Hence, the optimal prices are $p=rz\text{-}d$ and $v=d/2$.

Proof of Lemma 5.4

The base producer's profit from selling the base-software and n applications is continuous in n and his profit when $n=0$ is given by $2(rz_H\text{-}d)>0$.

$$\frac{\partial \Pi_B(n)}{\partial n} = \begin{cases} \frac{r^2 z_L}{2c} + \frac{d^2}{2cz_H} - \alpha d & \text{if } n \le 2c/r \\ 2cz_L/n^2 + \frac{d^2}{2cz_H} - \alpha d & \text{if } 2c/r < n \le 2cz_H/d \\ 2c(z_L + z_H)/n^2 - \alpha d & \text{if } n > 2cz_H/d \end{cases}$$

The base producer's profit is linear in n for $n < 2c/r$. If the profit is decreasing for $n < 2c/r$, then it is decreasing for every value of $n > 0$, because $2cz_L/n^2 < r^2z_L/(2c)$ if and only if $n > 2c/r$, and $2cz_H/n^2 < d^2/(2cz_H)$ iff $n > 2cz_H/d$. Hence, if $\alpha d > \frac{r^2z_L}{2c} + \frac{d^2}{2cz_H}$ then the optimal n is zero.

If the profit is increasing in n for $n < 2c/r$ and is decreasing in n when $n = 2cz_H/d$, then the optimal n is given by the first order condition in the range $2c/r < n < 2cz_H/d$. That is, if $\frac{d^2(z_L+z_H)}{2cz_H} < \alpha d < \frac{r^2 z_L}{2c} + \frac{d^2}{2cz_H}$ the optimal n is given by $\sqrt{\frac{4c^2 z_L z_H}{d(2cz_H\alpha - d)}}$.

If the profit is increasing in n when $n = 2cz_H/d$, then the optimal n is given by the first order condition in the range $n > 2cz_H/d$. That is, if $\alpha d < \frac{d^2(z_L+z_H)}{2cz_H}$ the optimal n is given by $\sqrt{\frac{2c(z_L+z_H)}{\alpha d}}$.

Chapter 6
Contribution, Limitations and Future Research

In this dissertation I develop three game theoretic models, that capture the interactions between buyers and sellers in B2C, B2B and software markets, when sellers can utilize multiples selling channels, and hence need to understand the relationships between demands on the different channels. Here, rather than cover specific results for each of the models, which were already discussed at the end of each relevant chapter, I would summarize the main contributions, limitations and possibilities for future research related to each of the three models.

In the first model, presented in Chapter 3, I examine the simultaneous use of auctions and posted price for selling consumer goods online. It is the first research to examine how a seller should design such a dual channel to maximize profit, under which conditions the dual channel outperforms the single channel, and the importance of designing the two channels jointly. Our work is innovative since we model how consumers, with stochastic valuations for the item and stochastic arrival times, choose between the two channels. We find a weakly dominant bidding strategy and the unique symmetric equilibrium, given by a threshold, for the auction participation strategy.

There are a number of interesting areas for future research. By parameterizing our model on R, the seller's reserve price, we could identify the optimal reserve price as well as the posted price, lot size and auction length. We suspect that the lot-size and auction length variables already capture most of the effects of the reserve price. Such a result would be interesting in its own right. The heuristic we have proposed in Proposition 3.2 is optimistic in its assessment of the bidder's ability to estimate the auction discount. Future research could explore alternative heuristics and their impact on sellers' decisions. Our model could also be extended to situations in which there is a finite supply that must be allocated between the two channels. Another extension would introduce competition between selling channels to address situations in which there are multiple auctioneers and posted price sellers in the market. In all of these possible extensions the underlying interaction between the auction lot size, auction length, and posted price introduced in this research will play an important role. As demonstrated in Figures 3.6 and 3.7, failure to manage their interactions correctly can significantly reduce revenues.

In chapter 4, I model the interaction between different supplier types on B2B spot market. To the best of our knowledge this is the first research to model a spot market with two types of suppliers: a supplier who faces contracted demand with fixed unit price, and a supplier who works solely on the spot market. Advances in communication networks, and the widespread use of the Internet, enable new online spot markets to proliferate in industries that used to be dominated by forward contracting. Hence, suppliers that have forward contracts face new managerial decisions: Should they invest in a new spot market? Would they be better off as spot market's demand increases, given that other suppliers work solely on the spot market? Can they contract for a lower unit price due to the existence of the spot market and under which conditions should they give priority to contracted demand, rather than sell their inventory on the spot market? Our research provides guidelines and answers to these questions.

Our model allows for either positive or negative correlation between contracted demand and spot market demand. We show that total industry supply and spot market supply are higher with negative correlation, and that both buyer-firms and suppliers that have forward contracts can benefit from negative correlation. However, suppliers that work only on the spot market would be better off working in industries where contracted demand and spot market demand are positively correlated.

One limitation of our model is using the Bernoulli distribution of contracted demand. Future work can examine the validity of our results with different distributions. Another extension can examine what happen when suppliers have different production costs or when there are different numbers of suppliers from each type. In this dissertation I focus on the only equilibrium in which the supplier with contracts can not satisfy a high level of contracted demand, and name it the Liquidation Equilibrium. Clearly, the analysis of other equilibria, in which the supplier with contracts can satisfy high contracted demand or sells units on the spot market before contracted demand is fully satisfied, can also compliment our work.

In the third model, presented in chapter 5, I focus on the software industry and the complementary relationship between base-software and applications. The model is innovative, since it captures the tradeoffs user-firms face when choosing between in-house development of business application and using packaged applications. In this model user-firms are heterogeneous in application needs and operation scale, and so I can analyze the effect of firms' size (operation scale) on firms' choices and on the software producers' profits and strategy. It is the first research to examine channel coordination and value of integration in this unique setting.

There are several possible extensions. First, in this research I consider only the two extreme strategies – all applications owned by the base-software producer or all applications sold by ISVs. Hence, I do not model channel conflict and competition between base producer and ISVs when the base-software producer enters the applications market and also supports ISVs entry. Another extension is to consider learning effects and reusability of software code when several applications are developed by the same firm.

All three models show that in industries with multiple channels, sellers have to re-evaluate their operational decisions, and that understanding the tradeoffs buyers face when choosing between channels, or the relationship between demands on the different channels, is necessary for optimizing the use of multiple channels.

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Appendix

Additional numerical results for Chapter 3

Tables A.1 and A.2 present the optimal design of the dual channel, when the participation threshold used by high valuation consumers is derived from Proposition 3.2, for $H = 5$ days and $H = 3$ days (the base case of $H = 7$ days is in §3.3). The results are very similar to those in Table 3.3 except that the auctions are shorter, with smaller lot sizes, and the posted prices are lower.

λ : w:	$0.1/day	$0.5/day	$1/day	$2/day	$3/day	$4/day	$5/day
0.5/day	No auction, p=$50						
1/day	1; H; $52	1; H; $53	1; H; $53	1; H; $54	1; H; $54	1; H; $54	1; H; $55
2/day	1; H; $52	1; H; $52	1; H; $52	1; H; $52	1; H; $53	1; H; $53	2; H; $55
5/day	1; 53; $52	1; 55; $52	3; H; $52	4; H; $53	5; H; $54	5; H; $55	5; H; $55
10/day	1; 27; $52	1; 27; $52	1; 28; $52	10; H; $53	11; H; $54	12; H; $55	13; H; $57
20/day	1; 13; $52	1; 14; $52	1; 14; $52	22; H; $53	24; H; $54	26; H; $56	27; H; $57
30/day	1; 9; $52	1; 9; $52	1; 9; $52	35; H; $53	39; H; $55	40; H; $56	41; H; $57
40/day	1; 7; $52	1; 7; $52	1; 7; $52	48; H; $53	53; H; $55	54; H; $56	56; H; $57
50/day	1; 5; $52	1; 5; $52	1; 5; $52	62; H; $53	66; H; $55	68; H; $56	71; H; $58
60/day	1; 4; $52	1; 4; $52	1; 4; $52	75; H; $53	80; H; $55	82; H; $56	86; H; $58

Table A.1: The optimal (q, T [hr], p [$\$$]) for various parameter values for H = 5 days and $\bar{t}$ derived from Proposition 3.2.

λ : w:	$0.1/day	$0.5/day	$1/day	$2/day	$3/day	$4/day	$5/day
0.5/day	No auction, p=$50						
1/day	No auction, p=$50						
2/day	1; H; $52	1; H; $53	1; H; $53	1; H; $53	1; H; $53	1; H; $53	1; H; $54
5/day	1; 53; $52	1; 55; $52	2; H; $52	2; H; $52	2; H; $53	2; H; $53	2; H; $53
10/day	1; 27; $52	1; 27; $52	1; 28; $52	4; H; $52	5; H; $53	5; H; $53	6; H; $54
20/day	1; 13; $52	1; 14; $52	1; 14; $52	11; H; $52	12; H; $53	13; H; $53	13; H;$54
30/day	1; 9; $52	1; 9; $52	1; 9; $52	1; 9; $52	19; H; $53	21; H; $54	21; H;$54
40/day	1; 7; $52	1; 7; $52	1; 7; $52	1; 7; $52	27; H; $53	29; H; $54	30; H;$55
50/day	1; 5; $52	1; 5; $52	1; 5; $52	1; 5; $52	35; H; $53	37; H; $54	39; H;$55
60/day	1; 4; $51	1; 4; $52	1; 4; $52	1; 4; $52	42; H; $53	45; H; $54	47; H;$55

Table A.2: The optimal (q, T [hr], p [$\$$]) for various parameter values for H = 3 days and $\bar{t}$ derived from Proposition 3.2.

Results with more knowledgeable consumers for Chapter 3

We assume that consumers evaluate the expected auction discount as if the total number of bidders of each type is given by the expected value of the relevant Poisson arrival process. We use this assumption to derive the suggested functional form for the threshold used by high valuation consumers. We believe this assumption to be more realistic than to expect that consumers can accurately determine the expected discount from the auction using the Poisson distribution and Equation 3.5. To examine the effect of this assumption on our results, we look at the optimal design of the dual channel when consumers are so insightful that they evaluate the auction discount using the Poisson distribution of the number of bidders. That is, when high valuation consumers derive their participation threshold directly from Equation 3.5 by solving

$\Pr(win|p)p - E[auction\_payment \mid p] = w\bar{t}$, where

$$p\Pr(win \mid p) - E[auction\_payment \mid p] =$$

$$p\left(\sum_{x=0}^{q-1}\sum_{y=q-x}^{\infty}\frac{e^{-\lambda_1 T}(\lambda_1 T)^y}{y!}\frac{e^{-\lambda_2 T}(\lambda_2 T)^x}{x!}+\sum_{x=q}^{\infty}Q(x)\sum_{y=0}^{\infty}\frac{e^{-\lambda_1 T}(\lambda_1 T)^y}{y!}\frac{e^{-\lambda_2 T}(\lambda_2 T)^x}{x!}+\sum_{x=0}^{q-1}\sum_{y=0}^{q-x-1}\frac{e^{-\lambda_1 T}(\lambda_1 T)^y}{y!}\frac{e^{-\lambda_2 T}(\lambda_2 T)^x}{x!}\right)-$$

$$\left(\begin{array}{l}\displaystyle\sum_{x=0}^{q-1}\sum_{y=q-x}^{\infty}\frac{e^{-\lambda_1 T}(\lambda_1 T)^y}{y!}\frac{e^{-\lambda_2 T}(\lambda_2 T)^x}{x!}O(q-x,y)+\sum_{x=q}^{\infty}pQ(x)\sum_{y=0}^{\infty}\frac{e^{-\lambda_1 T}(\lambda_1 T)^y}{y!}\frac{e^{-\lambda_2 T}(\lambda_2 T)^x}{x!}+\\ \displaystyle R\sum_{x=0}^{q-1}\sum_{y=0}^{q-x-1}\frac{e^{-\lambda_1 T}(\lambda_1 T)^y}{y!}\frac{e^{-\lambda_2 T}(\lambda_2 T)^x}{x!}\end{array}\right)=$$

$$\begin{array}{l}\displaystyle =\sum_{x=0}^{q-1}\sum_{y=q-x}^{\infty}\frac{e^{-\lambda_1 T}(\lambda_1 T)^y}{y!}\frac{e^{-\lambda_2 T}(\lambda_2 T)^x}{x!}\left(p-\left(\underline{v}+(p-\underline{v})\frac{y-(q-x-1)}{y+1}\right)\right)\\ \displaystyle +(p-R)\sum_{x=0}^{q-1}\sum_{y=0}^{q-x-1}\frac{e^{-\lambda_1 T}(\lambda_1 T)^y}{y!}\frac{e^{-\lambda_2 T}(\lambda_2 T)^x}{x!}\end{array} \qquad \text{(A.1)}$$

Tables A.3 to A.5 present the optimal design of the dual channel when the participation threshold used by high valuation consumers is derived directly from Equation 3.5, for different values of H (the upper limit of the auction length). When consumers are as knowledgeable as the seller, the optimal auction length remains the longest possible, H time units, even for small w values. For one-unit auctions, the threshold found directly from Equation 3.5 is significantly larger than the threshold found based on Proposition 3.2. This means that on average, when consumers are knowledgeable, more high valuation consumers will participate in the auction. As a result, most of the auction sales will be going to the class of buyers that the seller wants buying at the posted price. The only way the seller can increase revenue by adding auctions with small lots is to use long auctions in order to increase the expected auction price and deter the high valuation consumers. For long auctions with large lots, the difference between these two thresholds (the threshold found based on Proposition 3.2, and the threshold found directly from Equation 3.5) decreases as w and λ increase. Thus, for large values of w and λ, the two thresholds yield the same optimal design. For small values of w and λ, however, although short one-unit auctions perform very well in the first case (when the participation threshold is given by Proposition 3.2) they result in a decrease in revenue (compared to the posted price channel alone) in the second case (when the participation threshold is derived from Equation 3.5). Using the optimal designs listed in Tables A.3 - A.5, the seller can still increase his revenue by adding the auctions, but the increase is smaller than when consumers are less knowledgeable.

λ : w:	$0.1/day	$0.5/day	$1/day	$2/day	$3/day	$4/day	$5/day
0.5/day	No auction , p=$50						
1/day	No auction , p=$50		1; H; $52	1; H; $53	1; H; $53	1; H; $54	1; H; $54
5/day	4; H; $50	5; H; $51	6; H; $52	7; H; $54	8; H; $55	9; H; $57	9; H; $58
10/day	10; H; $50	13; H; $51	15; H; $52	16; H; $54	18; H; $56	19; H; $58	19; H; $59
20/day	25; H; $50	30; H; $51	32; H; $52	36; H; $54	38; H; $56	39; H; $58	40; H; $59
30/day	41; H; $50	47; H; $51	49; H; $52	56; H; $55	57; H; $56	59; H; $58	62; H; $60
40/day	55; H; $50	62; H; $51	68; H; $52	75; H; $55	79; H; $57	79; H; $58	83; H; $60
50/day	68; H; $50	80; H; $51	87; H; $52	93; H; $55	97; H; $56	100; H; $58	103; H; $60
60/day	81; H; $50	93; H; $51	101;H;$52	112;H;$55	117; H; $57	119; H; $58	125; H; $60

Table A.3: The optimal (q, T [hr], p [$\$$]) for various parameter values for H = 7 days and $\bar{t}$ derived from Equation 3.5.

λ : w:	$0.1/day	$0.5/day	$1/day	$2/day	$3/day	$4/day	$5/day
1/day	No auction, p=$50					1; H; $54	1; H; $54
5/day	2; H; $50	3; H; $51	3; H; $51	4; H; $52	5; H; $54	5; H; $55	5; H; $55
10/day	6; H; $50	8; H; $51	9; H; $51	10; H; $53	11; H; $54	12; H; $55	13; H; $57
20/day	16; H; $50	19; H; $51	22; H; $52	23; H; $53	25; H; $55	26; H; $56	27; H; $57
30/day	26; H; $50	32; H; $51	34; H; $52	37; H; $53	39; H; $55	40; H; $56	41; H; $57
40/day	37; H; $50	41; H; $51	46; H; $52	49; H; $53	53; H; $55	55; H; $56	56; H; $57
50/day	45; H; $50	54; H; $51	59; H; $52	63; H; $53	67; H; $55	68; H; $56	69; H; $57
60/day	58; H; $50	66; H; $51	72; H; $52	76; H; $53	80; H; $55	81; H; $56	85; H; $58

Table A.4: The optimal (q, T [hr], p [$\$$]) for various parameter values for H = 5 days and $\bar{t}$ derived from Equation 3.5.

λ : w:	$0.1/day	$0.5/day	$1/day	$2/day	$3/day	$4/day	$5/day
1/day	No auction, p=$50						
5/day	1; H; $50	1; H; $50	1; H; $51	2; H; $51	2; H; $52	2; H; $52	2; H; $53
10/day	2; H; $50	3; H; $50	4; H; $51	5; H; $52	5; H;52	6; H; $53	6; H; $54
20/day	8; H; $50	9; H; $50	11; H; $51	12; H; $52	13; H;53	13; H; $53	14; H; $54
30/day	13; H; $50	15; H; $50	17; H;51	19; H; $52	20; H;53	22; H;54	22; H;54
40/day	20; H; $50	23; H; $50	24; H; $51	27; H; $52	28; H; $53	29; H; $54	30; H; $55
50/day	24; H; $50	29; H; $50	32; H;51	34; H;52	36; H;53	38; H;54	39; H;55
60/day	32; H; $50	35; H; $50	38; H; $51	40; H; $52	43; H; $53	46; H; $54	47; H; $55

Table A.5: The optimal (q, T [hr], p [$\$$]) for various parameter values for H = 3 days and $\bar{t}$ derived from Equation 3.5.

Managing the two channels independently

Table A.6 shows the suboptimal channels' design that would result if the two channels were managed independently. The auction length and lot size are chosen to maximize revenue per unit time when all consumer have low valuations; i.e., there is no posted price option. This is done by solving the following:

$$Max_{q,\,T}\; q\sum_{N=q+1}^{\infty} O\{q+1,N\}\frac{e^{-\lambda T}(\lambda T)^N}{N!}+R\sum_{N=0}^{q} N\frac{e^{-\lambda T}(\lambda T)^N}{N!}.$$

The result is that the auction design is independent of the waiting costs, and lot sizes tend to be larger than in Table 3.3. The posted price is set as if there is no auction channel: $Max(\underline{v},\bar{v}/2)$, the solution to $\text{Max}\,\lambda\Pr(V \geq p)p = \lambda(\bar{v}-p)/(\bar{v}-\underline{v})p$. The result is a lower price than optimal, and too many high valuation consumers purchase in the auction, leading to too few posted price sales.

λ : w:	$0.1/day	$0.5/day	$1/day	$2/day	$3/day	$4/day
1/day	3; H; $50	3; H; $50	3; H; $50	3; H; $50	3; H; $50	3; H; $50
2/day	7; H; $50	7; H; $50	7; H; $50	7; H; $50	7; H; $50	7; H; $50
5/day	17; H; $50	17; H; $50	17; H; $50	17; H; $50	17; H; $50	17; H; $50
10/day	34; H; $50	34; H; $50	34; H; $50	34; H; $50	34; H; $50	34; H; $50
20/day	69; H; $50	69; H; $50	69; H; $50	69; H; $50	69; H; $50	69; H; $50
30/day	104; H; $50	104; H; $50	104; H; $50	104; H; $50	104; H; $50	104; H; $50
40/day	139; H; $50	139; H; $50	139; H; $50	139; H; $50	139; H; $50	139; H; $50

Table A.6: The optimal (q, T [hr], p [$\$$]) for various parameter values when the two channels are managed independently.